

Class example, September 28, 2012

The following example was covered very quickly at the end of the morning lecture. Here is a more complete solution.

**Problem :**

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is a tautology by applying a series of equivalences to derive T.

**Solution:**

|   |                           |
|---|---------------------------|
| $(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$ | Law of Implication        |
| $\equiv (\neg p \vee \neg q) \vee (p \vee q)$                                 | DeMorgan's Law            |
| $\equiv ((\neg p \vee \neg q) \vee p) \vee q$                                 | Associative Law           |
| $\equiv (p \vee (\neg p \vee \neg q)) \vee q$                                 | Commutative Law           |
| $\equiv ((p \vee \neg p) \vee \neg q) \vee q$                                 | Associative Law           |
| $\equiv (T \vee \neg q) \vee q$   | Negation Law for $\vee$   |
| $\equiv T \vee q$   | Domination Law for $\vee$ |
| $\equiv T$  | Domination Law for $\vee$ |