University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2012

November 13, 2012

Homework 8, Due Wednesday, November 21, 2012

Problem 1:

A relation R is called *circular* iff $(c, a) \in R$ whenever $(a, b) \in R$ and $(b, c) \in R$. Prove that for any reflexive relation R, R is circular if and only if R is both symmetric and transitive.

Problem 2:

Let R be the relation on pairs of positive integers, $\mathbb{Z}^+ \times \mathbb{Z}^+$ given by $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Prove that R is reflexive, symmetric and transitive.

Problem 3:

A directed graph is called *acyclic* iff if it does not contain a directed cycle (a non-empty path from a vertex to itself). Show that for every directed acyclic graph G, the transitive-reflexive closure of the relation R represented by G is antisymmetric.

Problem 4:

Let R be the relation defined on $\{1, 2, 3, 4, 5\}$ that consists of $\{(1, 2), (2, 3), (3, 4), (4, 5), (5, 2)\}$.

a) Compute the matrix representation of R^2 using matrix multiplication; show your work.

b) Give the graph representation of the Transitive-Reflexive closure of R.

Problem 5:

[For this problem, use the lecture slides as the source of information on relational algebra.] A relational database contains information about Employees, their managers, and about how they share rides to work:

`Empolyee`

E_ID	Name	Date_Hired
233	Alice	1999
300	Bob	2011
440	Carol	2008
004	Dave	1999
888	Bob	1977

`Manages`

E_ID_mngr	E_ID_emp
233	300
233	888
300	004
004	440

`ShareRide`

E_ID_driver	E_ID_passanger
440	888
004	233

- Indicate the key attribute in the table `Employee`.

- For each of the two relationships `Manages` and `ShareRide` indicate whether it is one-one, many-one, one-many, or many-many.
- Write a Relational Algebra expression that returns the names of all employees hired during or after 1998.
- Write a Relational Algebra expression that returns the names of all employees who manager was hired later than they were hired.
- Write a Relational Algebra expression that returns the employee IDs of all employees who drive their managers to work.

Problem 6:

Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.

a) The set of all binary strings that start with 0 and have even length, or start with 1 and have odd length.

b) The set of all binary strings that have a 1 in every odd-numbered position counting from the start of the string.

Problem 7:

Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.

a) The set of all binary strings that contain at least two 0's and at most one 1.

b) The set of all binary strings that don't contain 110.

Extra Credit 8:

Give a state diagram for a (deterministic) finite state machine that recognizes the set of all binary strings that represent integers that are multiples of 3 when read from left to right. Indicate the start and final states in your diagram and clearly label each state. In addition to the diagram document your design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.