University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2012

November, 2012

Homework 6, Due Wednesday, November 7, 2012

In problems 1 and 2, f_n is the nth Fibonacci number where $f_0 = 0$, $f_1 = 1$ and $f_k = f_{k-1} + f_{k-2}$ for $k \geq 2$.

Problem 1:

Prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ when n is a positive integer.

Problem 2:

Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

prove that

$$A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$$

when n is a positive integer.

Problem 3:

Give a recursive definition of

a) The set of integers that are congruent to 1 or 3 modulo 7.

b) The set of polynomials in x with integer coefficients.

Problem 4:

Give a recursive definition of the set of bit strings that have the same number of zeros and ones.

Problem 5:

Give a recursive definition of the following set of ordered pairs of positive integers:

$$S = \{(a, b) \mid a \in Z^+, b \in Z^+, \text{and } a + b \text{ is odd}\}$$

Extra Credit 6:

The following two mystery sets of positive integers A and B are defined recursively as follows:

- $0 \in A$ and $1 \in B$.
- If $\lfloor n/2 \rfloor \in A$ and n is even then $n \in A$; if $\lfloor n/2 \rfloor \in A$ and n is odd then $n \in B$.
- If $\lfloor n/2 \rfloor \in B$ and n is even then $n \in B$, if $\lfloor n/2 \rfloor \in B$ and n is odd then $n \in A$.

Find the sets A, B. You need to give a formal, non-recursive description of these sets, for example you might say "A is the set of all prime numbers, and B is the set of all numbers divisible by 7" (not the real answer).