University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2012

October 28, 2012

Homework 5, Due Wednesday, October 31st, 2012

Problem 1:

Compute the GCD of 89 and 144 using the Euclidean Algorithm. Show the intermediate values that are computed. Do you recognize them?

Problem 2:

Prove that for every integer n, there are n consecutive composite integers. [Hint: Consider the n consecutive integers starting with $(n+1)!+2$.]

Problem 3:

Determine modular inverses for the following. Use the Euclidian algorithm to find the inverses. The inverses you give should be postive:
a) Find an inverse of 4 modulo 9.
b) Find an inverse of 5 modulo 14.
c) Find an inverse of 5 modulo 26.

Problem 4:

How many zeros are at the end of 200!. Justify your answer without computing 200!.

Problem 5:

Prove that for every positive integer n,

$$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2.$$

Problem 6:

Prove that 3 divides $n^3 + 2n$ when n is a positive integer.

Problem 7:

Let x be any fixed real number with $x \geq -1$. Prove that $(1+x)^n \geq 1 + nx$ for every integer $n \geq 0$.

Extra Credit 8:

Two integers a and b are *relatively prime* if and only if $gcd(a, b) = 1$. Consider any $n+1$ numbers between 1 and $2n$ (inclusive). Show that some pair of them are relatively prime.