University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2012

October 6, 2012

Homework 3, Due Wednesday, October 17, 2012

Problem 1:

Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.
a) $\exists x \forall y (x + y = y)$
b) $\exists x \exists y (((x \leq 0) \wedge (y \leq 0)) \wedge (x - y > 0))$

Problem 2:

Rewrite each of these statements so that the negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
a) $\neg \exists y \exists x P(x, y)$
b) $\neg \exists y (Q(y) \wedge \forall x \neg R(x, y))$
c) $\neg \exists y (\exists x R(x, y) \vee \forall x S(x, y))$

Problem 3:

Show that the premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, and $\neg s$ imply the conclusion that $q \to r$ using the inference rules and equivalences. How many rows would you need if you tried to do this using a truth table?

Problem 4:

Use rules of inference to show that if $\forall x (P(x) \vee Q(x))$ and $\forall x ((\neg P(x) \wedge Q(x)) \to R(x))$ are true then $\forall x (\neg R(x) \to P(x))$ is also true, where the domains of all quantifiers are the same.

Problem 5:

Prove or disprove: $n^2 + 3n + 1$ is always prime for integer $n > 0$.

Problem 6:

Suppose that $\mathcal{P}(A) = \mathcal{P}(B)$. Show that $A = B$.

Problem 7:

Suppose that $C \neq \emptyset$ and $A \times C = B \times C$. Show that $A = B$. Why do we need the assumption that $C \neq \emptyset$?

Problem 8:

Which, if any, of the following assumptions implies that $A = B$ for all sets A, B, and C? Justify your answers.

(a) $A \cup C = B \cup C$.

(b) $A \cap C = B \cap C$.

(c) Both $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

Extra Credit 9:

Five pirates, called Alan, Brian, Carl, Dan and Ed, found a treasure of 100 gold coins.

On their ship, they decide to split the coins using the following scheme. The first pirate in alphabetical order becomes the chief pirate. The chief proposes how to share the coins, and all *other* pirates (excluding the chief) vote for or against it. If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain. Thus, in the first round Alan is the chief: if his proposal is rejected, he is thrown overboard and Brian becomes the chief, etc; if Alan, Brian, Carl, and Dan are thrown overboard, then Ed becomes the chief and keeps the entire treasure.

The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent, what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.