University of Washington October 16, 2012
Department of Computer Science and Engineering
CSE 311, Autumn 2012

Homework 2, Due Wednesday, October 10, 2012

Problem 1:
Use truth tables to represent the values of each of these boolean functions:
a) $F(x, y, z) = \overline{x}yz + \overline{(xyz)}$
b) $F(x, y, z) = x(yz + \overline{y}\ \overline{z})$

Problem 2:
Find the sum-of-products expansion of these Boolean functions:
a) $F(x, y, z) = (x + z)y$
b) $F(x, y, z) = x$
c) $F(x, y, z) = x\overline{y}$

Problem 3:
Construct a circuit that computes the product of the two two-bit integers $(x_1x_0)_2$ and $(y_1y_0)_2$. The circuit should have four output bits for the bits in the product.

Problem 4:
Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.
a) $\forall x(R(x) \wedge H(x)$
b) $\exists x(R(x) \to H(x))$

Problem 5:
Let $C(x)$ be the statement "x has a cat", $D(x)$ be the statement "x has a dog", and $F(x)$ be the statement "x has a ferret". Express each of the following statements using quantifiers, logical connectives, and the above statements. The domain consists of all students in the class.
a) All students in the class have a cat, a dog, and a ferret.
b) Some student in the class has a cat and a ferret, but not a dog.
c) For each of the three animals, cats, dogs, and ferrets, there is a student in the class who has one of these animals as a pet.

Problem 6:
Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
a) All students in your class can solve quadratic equations.
b) Some student in your class does not want to be rich.

Problem 7:

Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent.

Problem 8:

Let $F(x, y)$ be the statement "x can fool y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements:
a)Everybody can fool Fred.
b)Everybody can fool somebody.
c)No one can fool himself or herself.

Extra Credit 9:

Design a boolean circuit that has six inputs, and one output where the output is 1 if exactly three of the inputs are 1, and is 0 otherwise. Your circuit should use as few gates as possible. Provide a brief explanation as to how your circuit works. As a starting point, the brute force approach, which looks at every possible way of having exactly three true inputs takes 1 OR gate, 20 AND gates, and 6 NOT gates. We don't know what the best possible result is, One instructor came up with a circuit that uses 13 AND gates, 3 XOR gates, 1 OR gate, and 6 NOT Gates.