University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2012

September 21, 2012

Homework 1, Due Wednesday, October 3, 2012

Problem 1:

Write each of these statements in the from "if p, then q" in English. [Hint: refer to the list of common ways to express conditional statements provided in 1.1 of the text.]

1. To be a citizen of this country, it is sufficient that you were born in the United States.
2. The Red Wings will win the Stanley Cup if their goalie plays well.
3. The beach erodes whenever there is a storm.
4. You will reach the summit unless you begin your climb too late.

Problem 2:

When planning a party you want to know whom to invite. Among the people you would like to invite are three touchy friends. You know that if Alice attends, she will be unhappy if Bob is there, Bob will attend only if Carol will be there, and Carol will not attend unless Alice also does. Which combinations of these three friends can attend so as not to make someone unhappy?

Problem 3:

The NOR connective takes two propositions and evaluates to true when both propositions are false and evaluates to false otherwise. The NOR of p and q is written as $p \downarrow q$.

Show how to write propositional formulas *only using the NOR connective* that are equivalent to each of the following:

1. $\neg p$
2. $p \vee q$
3. $p \wedge q$
4. $p \rightarrow q$

Problem 4:

Using only AND gates, OR gates. and inverters (NOT gates), draw the diagram of a circuit with two inputs that computes the same function of its inputs that a single two-input XOR gate does.

Problem 5:

Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not equivalent.

Problem 6:

Find a compound proposition involving the propositional variables p, q, and r, that is true when exactly one of p, q and r are true, and is false otherwise.

Problem 7:

Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology by applying a series of equivalences to derive T.

Problem 8:

Show that the negation of an unsatisfiable compound proposition is a tautology and the negation of a compound proposition that is a tautology is unsatisfiable.

Extra Credit 9:

You have two memory registers, each with the same number of bits. You have an operation, `XOR(R1,R2)`, which takes two registers, `R1` and `R2`, performs bitwise $\oplus$ between them, and stores the result in `R1`. Show how you can swap the contents of the two registers using a sequence of `XOR`s without temporary memory registers. Explain why this works.