Name: ______________________________________

Student ID: ______________________________________

CSE 311 Winter 2011: Sample Midterm Exam

(closed book, closed notes except for 1-page summary)

Total: 100 points, 5 questions. Time: 50 minutes

Instructions:

1. Write your name and student ID on the first sheet (once you start, write your last name on all sheets). Write or mark your answers in the space provided. If you need more space or scratch paper, additional sheets will be available. Make sure you write down the question number and your name on any additional sheets.
2. Tables for logical equivalence and set identities are included in the back.
3. Read all questions carefully before answering them. Feel free to come to the front to ask for clarifications.
4. *Hint 1*: You may answer the questions in any order, so if you find that you're having trouble with one of them, move on to another one that seems easier.
5. *Hint 2*: If you don't know the answer to a question, don't omit it - do the best you can! You may still get partial credit for whatever you wrote down. Good luck!

Do not start until you are given the "green signal"…

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1. (20 points: 4 points each) Circle True (T) or False (F) below. Very briefly justify your answers (e.g. by giving an example or counter-example, by citing a theorem or result we proved in class, or by *briefly* sketching a proof or construction).

a. For all propositional variables p and q, $(p \vee q) \rightarrow p$ is a tautology..................T F
Why/Why not?

b. $\exists n \in Z\ \forall m \in Z\ (nm = m)$ ....T F
Why/Why not?

c. In a vanpool of eight people, at least two were born on the same day of the week...T F
Why/Why not?

d. The function $f(x,y) = x-y$ from $Z \times Z$ to Z is a bijection...................................T F
Why/Why not?

e. For the hashing function $h(k) = k \bmod 8$, the keys 10 and 20 result in a collision……………………………………………………………………T F
Why/Why not?

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2. (20 points: 10 points each) Propositional Logic

a. Show that $\neg(p\leftrightarrow q) \equiv (p\wedge\neg q) \vee (q\wedge\neg p)$ using known logical equivalences (see tables at back of this exam).

b. Use a truth table to show that $\neg p\rightarrow(p\rightarrow q)$ is a tautology.

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3. (18 points: 10 and 8 points) Predicate Logic

a. Let S(x) be the predicate "x is a student," F(x) the predicate "x is a faculty member," and A(x,y) the predicate "x has asked y a question," where the domain is the set of all people in CSE. Use only the quantifiers $\exists$ and $\forall$ to express the following statements:

   i. There is a faculty member who has never been asked a question by any students.

   ii. All faculty members have been asked a question by at least two students.

b. Express the negations of each of these statements so that all negation symbols immediately precede predicates:

   i. $\forall x \exists y (P(x,y) \wedge \exists z R(x,y,z))$

   ii. $\forall x \exists y (P(x,y) \rightarrow Q(x,y))$

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4. (22 points: 12 and 10 points) Rules of Inference and Proofs

a. Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

b. Use a proof by contraposition to show that for all real numbers x, y, if $x + y \geq 2$, then $x \geq 1$ or $y \geq 1$.

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5. (20 points: 12 and 8 points) Sets and Number Theory

a. Let A and B be two sets such that $A \subseteq B$. Let U be the universal set.

i. Draw two Venn diagrams, one showing A and B, and another showing $\overline{A}$ and $\overline{B}$.

ii. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$.

b. State whether the following statements are TRUE or FALSE (no proofs needed):

i. If $n \mid x$, then x **mod** $n = 0$ for any integer x and any positive integer n.

ii. -7 **div** 3 = -2

iii. 11 ≡ 19 (mod 4)

iv. The number 8 is among the numbers generated by the linear congruential pseudorandom number generator: $x_{n+1} = 3x_n$ **mod** 11 with seed $x_0 = 2$.

END OF EXAM

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TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

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TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \lor q$
$p \to q \equiv \neg q \to \neg p$
$p \lor q \equiv \neg p \to q$
$p \land q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \land \neg q$
$(p \to q) \land (p \to r) \equiv p \to (q \land r)$
$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$
$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

TABLE 8 Logical Equivalences Involving Biconditionals.

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

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TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution