Final exam review

The final exam will be Monday, June 6, 2:30pm-4:20pm. You may bring two double-sided sheets of notes on 8 1/2" x 11" paper. Roughly 1/3 will be on material before the midterm and 2/3 on material since the midterm.

Below are a list of topics that you should know, along with suggested odd-numbered exercises if you want extra practice. The notation $[x.y : z_1, z_2, \ldots]$ means exercises $z_1, z_2, \ldots$ in section $x.y$. You don't have to do any of these problems if you feel confident about your knowledge of the material.

Chapter 1:

- Converting between propositions/predicates and English/mathematical statements. [1.1:9,17]
- Logical equivalences [1.2:27,33,57]
- Quantifiers [1.3:13,19,39,59,61; 1.4:7,13,23,25,31,43,49]
- Inference and logical equivalences [1.5:15,19,27]
- Proof strategies (direct, contrapositive, contradiction, cases, WLOG) [1.6:3,5,13,17,35,39; 1.7:3,5,13,15]

Chapter 2:

- sets
  - set builder notation [2.1:5]
  - subsets, empty set [2.1:7,9]. [2.1:15, but prove only using the definition of $\subseteq$ and not the set identities in Section 2.2]
  - power set [Define a function f whose domain is finite sets by $f(S) = P(S)$. Construct the inverse f^{-1}; i.e. a function such that $f^{-1}(f(S)) = S$ for any finite set S.]
  - Cartesian product [2.1:27,29]
  - intersection, union, complement, set difference: [2.2:19,25,29]
  - disjoint sets
  - cardinality and infinite sets. [Prove or disprove: If A and B are finite sets, then $|A \cup B| \leq |A| + |B|$.]
  - using set notation to understand other parts of the course: quantifiers, number theory, induction, etc.
- functions
  - terminology (domain/codomain/range/image/preimage). [2.3:5]
  - properties (onto/surjective, 1-1/injective, bijective). [2.3:19,21,31]
  - methods of proving a function has these properties (either directly or using the fact that function is increasing or decreasing). [2.3:23,35]

Chapter 3:

- divisibility [3.4:3,5,7]

- division algorithm [3.4:9ab]
- modular arithmetic [3.4:11,21,23]
- primes [3.5:9,35]
- relatively prime [3.5:11]
- FTA, GCD, LCM [3.5:27]
- multiplicative inverses, solving linear congruences [3.7:13,15]
- Fermat's little theorem [3.7:17]

Chapter 4:

- induction [4.1:15 (proving an equality), 21 (proving an inequality), 39 (proofs about sets), 49]
- strong and structural induction [4.2:3,13,29]
- variants of induction [4.2:25]
- recursive definitions [4.3:27,37]
- recursive algorithms [4.4:23]

Chapter 8:

- properties of relations (symmetry, antisymmetry, reflexivity, transitivity) [8.1:5,7]
- equivalence relations and partitions [8.5:3,15,45]
- partial orders [8.6:1,3,5,37]

Chapters 9 and 10:

- graph types (directed/undirected, self-loops/multiple-edges allowed or not, weighted or unweighted) [9.1:3,5,7,9 (but don't worry about the table),11]
- degree [9.2:15]
- paths, circuits and connectedness in undirected graphs [9.4:1, 27]
- tree definition [10.1:1]
- quantitative features of trees [10.1:15,23,37]

Chapter 11:

- Boolean algebra [11.1:3, 5a, 33]
- sum-of-products expansion [11.2:5]
- functional completeness [11.2:19]
- circuits [11.3:1,3, 11, 15]

Chapter 12:

- finite state machines with output [12.2: 1a, 3a, 9]

- finite state machines without output [12.3: 17, 27]
- nondeterministic finite automata and converting them to DFAs [12.3:51]
- regular sets and expressions [12.4:3,5,11]
- recognizing regular sets with NFAs [12.4:13]
- Turing machines [12.5:3,7,15]

Topics that will not be on the exam:

- truth tables
- rings and fields
- Chinese Remainder Theorem
- RSA
- triominoes
- induction on general well-ordered sets
- graph notions not listed above (e.g. strongly connected, Eulerian cycle, etc.)
- infinite graphs
- deciding whether to use weak/strong/structural induction (the question will specify which type of induction to use)