Name Student ID

CSE 311: Foundations of Computing I
Spring 2011
Midterm

Instructions

1. Start at the 1:30PM bell and stop at the 2:20PM bell.
2. There are 7 questions, worth a total of 100 points.
3. Write your name and student ID on the first sheet (once you start, write your last name on all sheets). Write or mark your answers in the space provided. If you need more space or scratch paper, additional sheets will be available. Make sure you write down the question number and your name on any additional sheets.
4. Tables for logical equivalences and inferences are included in the back.
5. You may bring in one double-sided sheet of notes. Apart from this, you may not use books, notes or calculators.
6. *Suggestion:* Skim all questions before starting, and answer the questions that seem easiest and fastest to answer first.
7. *Suggestion:* If you don't know the answer to a question, try to clearly write down what you know anyway, and you may still be able to get partial credit.

1. **Sets** *(12 points)*

   (a) Let A, B, C be sets. Express $A-(B-C)$ using only symbols from this list: $(,), \cap, \cup, A, B, C, \bar{A}, \bar{B}, \bar{C}$.

   (b) Prove or give a counter-example: for any sets A, B, $|P(A) \times P(B)| = |P(A \times B)|$. Here $P(S)$ is defined to be the power set of S, meaning $P(S) = \{T : T \subseteq S\}$.

2. **Quantifiers** *(12 points)*

   (a) Prove or disprove: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}(x + y = 0)$.

   (b) Prove or disprove: $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}(x + y = 0)$.

3. **Propositional logic** *(20 points)*

   (a) Show that $(\neg p \to q) \to p \equiv q \to p$ using logical equivalences from the table at the back of the exam. Use at most one equivalence per line.

   (b) Construct a truth table for $(p \vee q) \wedge (q \to p)$. Is this a tautology, contradiction or contingency? Briefly indicate why.

4. **Predicate logic** *(20 points)*

   (a) Let $A(x, y)$ be the predicate "x has read book y", and $B(x, y, z)$ be the predicate "x prefers book y over book z" where the domain of x is the set of all people, and the domain of y and z is the set of all books. Express the following statements using $\forall, \exists, \neg$.

      i. There is a book that no one prefers over any other book.

      ii. Anyone who has read any book has a book that they prefer over all other books.

   (b) Express the negations of each of the following statements in a way such that $\neg$ does not precede a $\forall, \exists$ or (.

      i. $\forall x(\exists y(A(x, y) \vee \exists z(A(x, z) \wedge \neg B(x, y, z))))$.

      ii. $\exists y(\forall x(A(x, y) \rightarrow \forall z((y \neq z) \rightarrow B(x, y, z))))$.

5. **Proof** *(12 points)* Suppose $x_1, x_2, x_3 \in \mathbb{R}$. Define the mean of these numbers to be

$$\bar{x} := \frac{x_1 + x_2 + x_3}{3}$$

   Prove that there exists $i \in \{1, 2, 3\}$ such that $x_i \geq \bar{x}$. *Hint: try a proof by contradiction.*

6. **Functions** *(12 points)* In each row, f is a function from $A \to B$. Mark Y/N to indicate whether f is surjective or injective. Briefly justify your answers.

A	B	f	surjective	injective
$\{0, 1, \ldots, 29\}$	$\mathbb{Z} \times \mathbb{Z}$	$f_1(x) = (x \bmod 5, x \bmod 6)$		
$\mathbb{R}^+$	$\mathbb{R}^+$	$f_2(x) = \sqrt{x}$		
$\mathbb{Z} \times \mathbb{Z}$	$\mathbb{Z} \times \mathbb{Z}$	$f_3(x, y) = (x, y - x^2)$		

7. **Modular arithmetic** *(12 points)*

   (a) Calculate $5^{256} \pmod 7$.

   (b) Find the smallest positive integer x satisfying $4x \equiv 3 \pmod 9$, if one exists, or write NONE, if none exists.

   (c) Find the smallest positive integer x satisfying $3x \equiv 4 \pmod 9$, if one exists, or write NONE, if none exists.

END OF EXAM

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

$\neg(p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditionals.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization