CSE 311: Foundations of Computing I
Sample midterm

1. **Prime factorization** *(4 points)* Compute the prime factorization of 320 and 450. Compute $\gcd(320, 450)$ and $\text{lcm}(320, 450)$, expressing your answer as a product of prime factors.

2. **Sets** *(12 points)* Let $A = \{z \in \mathbb{Z} | 1 \leq z \leq 8\}$, $B = \{z \in \mathbb{Z} | \, 2|z\}$ and let C be the set of primes. Fill in the table below with the set that each expression evaluates to:

Expression	Value
$A \cap B$	
$B \cap C$	
$A - (B \cup C)$	

3. **Primes and predicates** *(8 points)* Translate the expression "p is prime" into mathematical notation, i.e. using only $\forall, \exists, |, \mathbb{Z}$ and other mathematical symbols.

4. **Propositional logic** *(20 points)*
   (a) Show that $\neg(p \oplus q) \equiv (p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$ using logical equivalences from the table at the back of the exam and the fact that $p \oplus q \equiv \neg(p \leftrightarrow q)$.
   (b) Construct a truth table for $q \vee (p \rightarrow \neg q)$. Is this a tautology, contradiction or contingency? Briefly indicate why.

5. **Predicate logic** *(20 points)*
   (a) Let $S(x)$ be the predicate "x is a student", $R(y)$ the predicate "y is a road in Seattle" and $V(x, y)$ the predicate "x has visited road y." Express the following statements using $\forall, \exists, \neg$.
      i. No student has visited all roads in Seattle.
      ii. At least two roads were visited by all students.
      iii. *[0 points]* There exist two roads such that I could not travel both and I took the one less traveled by.
   (b) Express the negations of each of the following statements in a way such that $\neg$ does not precede a $\forall, \exists$ or (.
      i. $\exists x \forall y (P(x, y) \wedge \exists z \neg Q(x, z))$
      ii. $\forall x (P(x) \rightarrow \exists y (Q(x, y)))$.

6. **Proof** *(10 points)* If A and B are sets, then does $A - B = \emptyset$ imply that $A = B$? Prove, or give a counter-example.

7. **Proof** *(10 points)* Prove that $\forall a, b, n \in \mathbb{Z} \; (4|n \wedge n = ab) \rightarrow (2|a \vee 2|b)$.

8. **Functions** *(18 points)* In each row, f is a function from $A \rightarrow B$. Mark Y/N to indicate whether f is surjective or injective. Briefly justify your answers.

A	B	f	surjective	injective
$\mathbb{Z}$	{0,1,2}	$f(x) = x \bmod 3$		
$\mathbb{Z}$	$\mathbb{Z}^+$	$f(x) = \|x - 1\|$		
$\mathbb{Z} \times \mathbb{Z}$	$\mathbb{Z}$	$f(x, y) = 3x + 7y$		

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 8 Logical Equivalences Involving Biconditionals.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization