CSE 311 Foundations of Computing I

Lecture 20

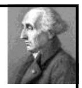
Relations, Graphs, Finite State Machines

Autumn 2011

Announcements

- Reading assignments
  - 7^{th} Edition, Sections 9.3 and 13.3
  - 6^{th} Edition, Section 8.3 and 12.3
  - 5^{th} Edition, Section 7.3 and 11.3

Lecture highlights



Let A and B be sets,
A binary relation from A to B is a subset of $A \times B$

$S \circ R = \{(a, c) \mid \exists\, b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$

$R^1 = R$; $R^{n+1} = R^n \circ R$

n-ary relations

Let $A_1, A_2, \ldots, A_n$ be sets. An n-ary relation on these sets is a subset of $A_1 \times A_2 \times \ldots \times A_n$.

Relational databases

Student_Name	ID_Number	Major	GPA
Knuth	328012098	CS	4.00
Von Neuman	481080220	CS	3.78
Von Neuman	481080220	Mathematics	3.78
Russell	238082388	Philosophy	3.85
Einstein	238001920	Physics	2.11
Newton	1727017	Mathematics	3.61
Karp	348882811	CS	3.98
Newton	1727017	Physics	3.61
Bernoulli	2921938	Mathematics	3.21
Bernoulli	2921939	Mathematics	3.54

Alternate Approach

Student_ID	Name	GPA
328012098	Knuth	4.00
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238082388	Russell	3.85
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1727017	Newton	3.61
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2921938	Mathematics
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Database Operations

Projection

Join

Select

Matrix representation

Relation R on A={a_1, … a_p}

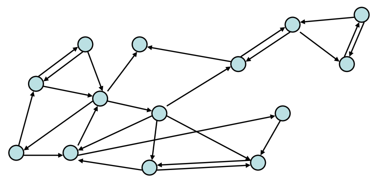
$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, a_j) \in R, \\ 0 \text{ if } (a_i, a_j) \notin R. \end{cases}$$

{(1, 1), (1, 2), (1, 4), (2,1), (2,3), (3,2), (3, 3) (4,2) (4,3)}

Directed graphs

G = (V, E)
- V – vertices
- E – edges, order pairs of vertices

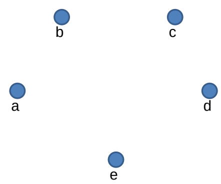
Path: v_1, v_2, …, v_k, with (v_i, v_{i+1}) in E
- Simple Path
- Cycle
- Simple Cycle



Representation of relations

Directed Graph Representation (Digraph)

{(a, b), (a, a), (b, a), (c, a), (c, d), (c, e) (d, e) }



Paths in relations

Let R be a relation on a set A. There is a path of length n from a to b if and only if $(a,b) \in R^n$

Connectivity relation

Let R be a relation on a set A. The connectivity relation R* consists of the pairs (a,b) such that there is a path from a to b in R.

$$R^* = \bigcup_{k=0}^{\infty} R^k$$

Properties of Relations

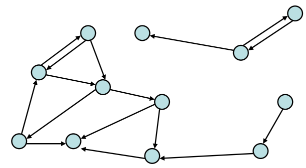
Let R be a relation on A

R is reflexive iff (a,a) ∈ R for every a ∈ A

R is symmetric iff (a,b) ∈ R implies (b, a)∈ R

R is transitive iff (a,b)∈ R and (b, c)∈ R implies (a, c) ∈ R

Transitive-Reflexive Closure



Add the minimum possible number of edges to make the relation transitive and reflexive

The transitive-reflexive closure of a relation R is the connectivity relation R*

Finite state machines

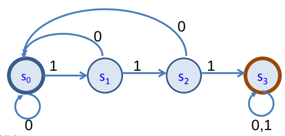
States

Transitions on inputs

Start state and finals states

The language recognized by a machine is the set of strings that reach a final state

State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3



What language does this machine recognize?