

CSE 311 Foundations of Computing I

Lecture 19
Relations
Autumn 2011

Announcements

- Reading assignments
 - 7th Edition, Section 9.1 and pp. 594-601
 - 6th Edition, Section 8.1 and pp. 541-548
 - 5th Edition, Section 7.1 and pp. 493-500
- Upcoming topics
 - Relations
 - Finite State Machines

Definition of Relations

Let A and B be sets,
A **binary relation from A to B** is a subset of $A \times B$

Let A be a set,
A **binary relation on A** is a subset of $A \times A$

Relation Examples

$$R_1 = \{(a, 1), (a, 2), (b, 1), (b, 3), (c, 3)\}$$

$$R_2 = \{(x, y) \mid x \equiv y \pmod{5}\}$$

$$R_3 = \{(c_1, c_2) \mid c_1 \text{ is a prerequisite of } c_2\}$$

$$R_4 = \{(s, c) \mid \text{student } s \text{ had taken course } c\}$$

Properties of Relations

Let R be a relation on A

R is **reflexive** iff $(a, a) \in R$ for every $a \in A$

R is **symmetric** iff $(a, b) \in R$ implies $(b, a) \in R$

R is **antisymmetric** iff $(a, b) \in R$ and $a \neq b$ implies $(b, a) \notin R$

R is **transitive** iff $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

Combining Relations

Let R be a relation from A to B
Let S be a relation from B to C
The composite of R and S, $S \circ R$ is the relation from A to C defined

$$S \circ R = \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}$$

Examples

$(a,b) \in \text{Parent}$: b is a parent of a

$(a,b) \in \text{Sister}$: b is a sister of a

What is $\text{Parent} \circ \text{Sister}$?

What is $\text{Sister} \circ \text{Parent}$?

$$S \circ R = \{(a, c) \mid \exists b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$$

Examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother express

Uncle: b is an uncle of a

Cousin: b is a cousin of a

Powers of a Relation

$$R^2 = R \circ R = \{(a, c) \mid \exists b \text{ such that } (a,b) \in R \text{ and } (b,c) \in R\}$$

$$R^0 = \{(a,a) \mid a \in A\}$$

$$R^1 = R$$

$$R^{n+1} = R^n \circ R$$

How is



related to



?

How is




related to



?

<http://genealogy.math.ndsu.nodak.edu/>

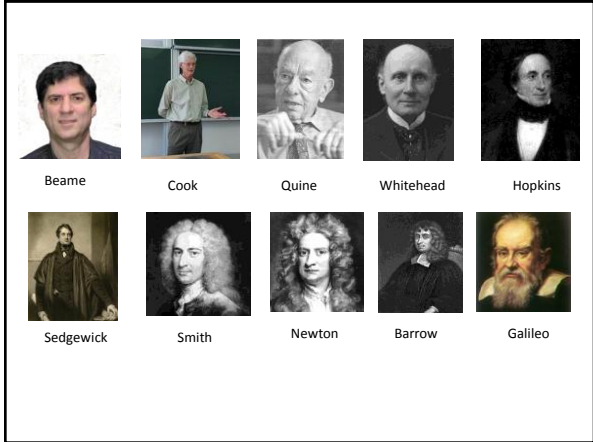
Mathematics Genealogy Project

Edward Delano Lazowska
[MamSched](#)
 Ph.D. University of Toronto 1977 

Dissertation: *Characterizing Service Time and Response Time Distributions in Queuing Network Models of Computer Systems*
 Advisor: [Kenneth Clem Sevick](#)

Students:
 Click [here](#) to see the students listed in chronological order.

Name	School	Year	Descendants
Thomas Anderson	University of Washington	1991	54
Robert Bedichuk	University of Washington	1994	
John Bennett	University of Washington	1988	9
Brian Bershad	University of Washington	1990	16
Jeffrey Chase	University of Washington	1995	7
Sung Chung	University of Washington	1990	
Edward Felten	University of Washington	1993	8
Richard Garner	University of Washington	1982	
Patricia Jacobson	University of Washington	1984	
Henry (Hank) Levy	University of Washington	1981	123
Yi-Bing Lin	University of Washington	1990	13



Nicolaus Copernicus	Galileo Galilei
Georg Rheticus	Vincenzo Viviani
Moritz Steinmetz	Issac Barrow
Christoph Meurer	Isaac Newton
Philipp Muller	Roger Cotes
Erhard Weigel	Robert Smith
Gottfried Leibniz	Walter Taylor
Noctas Malebranache	Stephen Whisson
Jacob Bernoulli	Thomas Postlethwaite
Johann Bernoulli	Thomas Jones
Leonhard Euler	Adam Sedgewick
Joseph Lagrange	William Hopkins
Jean-Baptiste Fourier	Edward Routh
Gustav Dirichlet	Alfred North Whitehead
Rudolf Lipschitz	Willard Quine
Felix Klein	Hao Wang
C. L. Ferdinand Lindemann	Stephen Cook
Herman Minkowski	Paul Beame
Constantin Caratheodory	
Georg Aumann	
Friedrich Bauer	
Manfred Paul	
Ernst Mayr	
Richard Anderson	

n-ary relations

Let A_1, A_2, \dots, A_n be sets. An n-ary relation on these sets is a subset of $A_1 \times A_2 \times \dots \times A_n$.

Relational databases

Student_Name	ID_Number	Major	GPA
Knuth	328012098	CS	4.00
Von Neuman	481080220	CS	3.78
Von Neuman	481080220	Mathematics	3.78
Russell	238082388	Philosophy	3.85
Einstein	238001920	Physics	2.11
Newton	1727017	Mathematics	3.61
Karp	348882811	CS	3.98
Newton	1727017	Physics	3.61
Bernoulli	2921938	Mathematics	3.21
Bernoulli	2921939	Mathematics	3.54

Alternate Approach

Student_ID	Name	GPA	Student_ID	Major
328012098	Knuth	4.00	328012098	CS
481080220	Von Neuman	3.78	481080220	CS
238082388	Russell	3.85	481080220	Mathematics
238001920	Einstein	2.11	238082388	Philosophy
1727017	Newton	3.61	238001920	Physics
348882811	Karp	3.98	1727017	Mathematics
2921938	Bernoulli	3.21	348882811	CS
2921939	Bernoulli	3.54	1727017	Physics
			2921938	Mathematics
			2921939	Mathematics

Database Operations

Projection

Join

Select