CSE 311 Foundations of Computing I

Lecture 16
Functions on Recursively Defined Sets and Structural Induction
Autumn 2011

Announcements

- Reading assignments
  - Today:
    - 5.3 7th Edition
    - 4.3 6th Edition
    - 3.4 5th Edition (not all there)
- Midterm Friday, Nov 4
  - Closed book, closed notes, no calculators, cell phones, etc.
  - Sample midterm questions available on website
  - Extra office hours Thursday
    - Richard Anderson: 4:30-5:30 in room CSE 503
    - Paul Beame: 3:30-4:30 in room CSE 403

Highlights from last lecture

- Recursively defined sets
  - *Basis step:* Some specific elements are in S
  - *Recursive step:* Given some existing named elements in S some new objects constructed from these named elements are also in S
- Structural Induction:
  1. By induction we will show that P(x) is true for every x in S
  2. Base Case: Show that P is true for all elements of S mentioned in the *Basis step*
  3. Inductive Hypothesis: Assume that P is true for some arbitrary values of each of the existing named elements mentioned in the *Recursive step*
  4. Inductive Step: Prove that P holds for each new element constructed in the *Recursive step* using the elements mentioned in the Inductive Hypothesis
  5. Conclusion: Result follows by induction

Strings

- An *alphabet* Σ is any finite set of characters.
- The set Σ^* of *strings* over the alphabet Σ is defined by
  - Basis: $\lambda \in \Sigma^*$ (λ is the empty string)
  - Recursive: if $w \in \Sigma^*$, $x \in \Sigma$, then $wx \in \Sigma^*$

Palindromes

- Palindromes are strings that are the same backwards and forwards
- Basis: λ is a palindrome and any $a \in \Sigma$ is a palindrome
- Recursive step: If p is a palindrome then apa is a palindrome for every $a \in \Sigma$

Function definitions on recursively defined sets

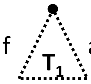
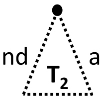
len (λ) = 0;
len (wa) = 1 + len(w); for $w \in \Sigma^*$, $a \in \Sigma$

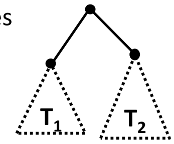
Reversal:
$\lambda^R = \lambda$
$(wa)^R = aw^R$ for $w \in \Sigma^*$, $a \in \Sigma$

Concatenation:
$w \bullet \lambda = w$ for $w \in \Sigma^*$
$w_1 \bullet w_2a = (w_1 \bullet w_2)a$ for $w_1, w_2 \in \Sigma^*$, $a \in \Sigma$

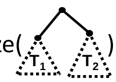
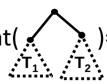
len(x•y)=len(x)+len(y) for all strings x and y

Rooted Binary trees

- Basis: ● is a rooted binary tree
- Recursive Step: If  and  are rooted binary trees then so is:



Functions defined on rooted binary trees

- size(●)=1
- size() = 1+size(T_1)+size(T_2)
- height(●)=0
- height()=1+max{height(T_1),height(T_2)}

For every rooted binary tree T size(T) ≤ $2^{\mathbf{height(T)+1}}$ -1