University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2011

November 20, 2011

Quiz Section, November 17, 2011: Selected answers

1. Composing relations:

   Recall: $S \circ R = \{(a,c) \mid \exists b \text{ s.t.} (a,b) \in R \text{ and } (b,c) \in S\}$

   We define the following relations:

   - $(a,b) \in$ Sibling: b is a's sibling
   - $(a,b) \in$ Mother: b is a's mother
   - $(a,b) \in$ Parent: b is a's parent
   - $(a,b) \in$ Daughter: b is a's daughter
   - $(a,b) \in$ Son: b is a's son
   - $(a,b) \in$ Child: b is a's child

   Use these relations to express the following:

   (a) $\{(a,c) \mid c$ is a's niece$\}$: $Daughter \circ Sibling$

   (b) $\{(a,c) \mid c$ is a's grandson$\}$: $Son \circ Child$

   (c) $\{(a,c) \mid c$ is a's grandmother$\}$: $Mother \circ Parent$

2. Proving relationship properties

   Prove that the relation R on a set A is symmetric if and only if $R = R^{-1}$.

   For an "if and only if" proof we need to prove both directions:

   (a) "only if' direction: Prove that if R is symmetric, then $R = R^{-1}$

   Assume that R is symmetric.
   To show that $R = R^{-1}$, we must show both directions:

   - Show that $R \subseteq R^{-1}$
     Let (x,y) be an arbitrary member of R. Then:
     $(y,x) \in R$ because R is symmetric
     $(x,y) \in R^{-1}$ by definition of inverse
     QED
   - Show that $R^{-1} \subseteq R$
     Let (x,y) be an arbitrary member of R^{-1}. Then:
     $(y,x) \in R$ by definition of inverse
     $(x,y) \in R$ because R is symmetric
     QED

   We have shown by direct proof that if R is symmetric then $R = R^{-1}$

(b) “if” direction: Prove that if $R = R^{-1}$, then R is symmetric

Assume $R = R^{-1}$. To show that R is symmetric, we must show that for any arbitrary (x, y) in R, (y, x) is also in R.

Let (x, y) be an arbitrary member of R.
$(y, x) \in R^{-1}$ by definition of inverse
$(y, x) \in R$ by assumption that $R = R^{-1}$
QED

We have shown by direct proof that if $R = R^{-1}$, then R is symmetric.

We have shown both the ”if” and ”only if” directions. Therefore, we have proven that the relation R on a set A is symmetric if and only if $R = R^{-1}$.