University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2011

November 20, 2011

Quiz Section, November 17, 2011

1. Relation representations

   For each of the relations below on the set {1,2,3,4},

   (a) Represent the relation as a matrix

   (b) Represent it as a directed graph

   - {(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)}
   - {(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)}
   - {(2,4), (4,2)}

2. Relation properties (Section 9.1 or 8.1: #3)
   For each of the relations in problem #1 above, decide if it is:

   (a) reflexive

   (b) symmetric

   (c) antisymmetric

   (d) transitive

3. Reflexive and symmetric closures, inverses
   For each of the relations in problem # 1 above, give a relation that is its:

   (a) reflexive closure

   (b) symmetric closure

   (c) inverse

4. Composing relations:

   Recall: $S \circ R = \{(a, c) \mid \exists b \text{ s.t.} (a, b) \in R \text{ and } (b, c) \in S\}$

   We define the following relations:

   - $(a, b) \in$ Sibling: b is a's sibling
   - $(a, b) \in$ Mother: b is a's mother
   - $(a, b) \in$ Parent: b is a's parent
   - $(a, b) \in$ Daughter: b is a's daughter
   - $(a, b) \in$ Son: b is a's son
   - $(a, b) \in$ Child: b is a's child

   Use these relations to express the following sets:

   (a) $\{(a, c) \mid c \text{ is } a\text{'s niece}\}$

   (b) $\{(a, c) \mid c \text{ is } a\text{'s grandson}\}$

   (c) $\{(a, c) \mid c \text{ is } a\text{'s grandmother}\}$

5. Powers of relations

   - $R^0 = \{(a, a) \mid a \in A\}$
   - $R^1 = R$
   - $R^2 = R \circ R$
   - $R^{n+1} = R^n \circ R$

   Find R^0, R^2, R^3, and R^4 for:

   - $R = \{(1, 2), (2, 3), (3, 1)\}$

6. Transitive Closure

   Draw the directed graph for R given in problem #5, above

   (a) Add edges to the graph until you have the reflexive closure of R.

   (b) Add more edges until you have the reflexive transitive closure of R.

   (c) What is the relationship between the final graph and the union of the sets found in probem #5: $R^0 \cup R^1 \cup R^2 \cup R^3 \cup R^4$?

   (d) Does R^4 contribute any edges?

7. The Connectivity Relation, Lemma 1 (section 8.4 or 9.4)

   The connectivity relation of R is defined as: $R^* = \bigcup_{k=0}^{\infty} R^k$

   - How would you describe this set?
   - Lemma 1: If there is a path in R from a to b, then there is such a path with length not exceeding n, where n is the number of vertices in the graph of R.
   - Prove Lemma 1
   - Because of Lemma 1, we have that:
     $R^* = \bigcup_{k=0}^{\infty} R^k$

8. Proving relationship properties

   Prove that the relation R on a set A is symmetric if and only if $R = R^{-1}$.