University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2011

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Quiz Section, October 27, 2011

Homework Review

1. Which, if any, of the following assumptions implies that $A = B$ for all sets A, B, and C?

   (a) $A \cup C = B \cup C$.
   (b) $A \cap C = B \cap C$.
   (c) Both $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

2. $E_7^3 = \{x \mid x \equiv 3 \pmod 7\}$ and $E_{21}^{10} = \{x \mid x \equiv 10 \pmod{21}\}$. Prove $E_{21}^{10} \subseteq E_7^3$.

New Stuff

1. Modular inverses

   (a) Find an inverse of 3 modulo 7
   (b) Find an inverse of 8 modulo 3
   (c) Find an inverse of 12 modulo 5

2. Prime factorization

   Find the prime factorization of the following:

   (a) 88
   (b) 10!

3. Induction

   (a) Seventh Edition: 5.1.11 Sixth Edition: 4.1.11.
      i. Find a formula for
      $$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$
      by examining the values of this expression for small values of n.
      ii. Prove the formula you conjectured in part (a).
   (b) Seventh Edition: 5.1.21 Sixth Edition: 4.1.21.
   Prove that $2^n > n^2$ if n is an integer greater than 4.
   (c) Seventh Edition: 5.1.31 Sixth Edition: 4.1.31.
   Prove using induction that 2 divides $n^2 + n$ whenever n is a positive integer.
   Bonus: Can you prove this result without using induction?

Template for proofs by induction:

"Proof"

1. *Define P(N) and b:*

   "By induction we will show that P(n) = _____ is true for every $n >=$ ____. "

2. *Prove P(b):*

   "Base Case: P(____) is true, because:

   "

3. *State the inductive hypothesis*

   "Inductive Hypothesis: Assume that P(k) is true for some arbitrary integer $k >=$ ____.

   That is, assume that _____. "

4. *Inductive Step: State the goal and then use the inductive hypothesis to reach it*

   "Inductive Step: We must show that if we assume P(k) is true, then P(k+1) is also true.

   That is, we must show that _____.

   Now show it. Use the goal to figure out what you need. Make sure you are using I.H. and point out where you are using it. (Dont assume $P(k+1)$):

   "

5. *Conclusion:*

   "We have completed the basis step and the inductive step. Therefore by mathematical induction, P(n) is true for all $n >=$ ____."