CSE311: Quiz Section, 10/6/2011

October 12, 2011

1. Possibly helpful tools on the textbook website, *www.mhhe.com/rosen* (7th Edition, "Student Edition")
   - Interactive Demonstration Applets
     - Truth Tables
     - Equivalences
   - Self Assessments
     - Conditional Statements
     - Quantified Statements
   - Guide to Writing Proofs
   - Common Mistakes

2. Prove that $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$ by rewriting with equivalences.

$$\begin{aligned}
(p \to r) \wedge (q \to r) &\equiv (p \vee q) \to r & \\
&\equiv \neg(p \vee q) \vee r & \text{Law of implication} \\
&\equiv (\neg p \wedge \neg q) \vee r & \text{DeMorgan's} \\
&\equiv (\neg p \vee r) \wedge (\neg q \vee r) & \text{Distributive} \\
&\equiv (p \to r) \wedge (q \to r) & \text{Law of implication}
\end{aligned}$$

3. Prove that $(p \wedge q) \to (p \to q)$ is a tautology by rewriting with equivalences.

$$\begin{aligned}
(p \wedge q) \to (p \to q) \equiv T & \\
\neg(p \wedge q) \vee (p \to q) & \quad \text{Law of implication} \\
\neg(p \wedge q) \vee (\neg p \vee q) & \quad \text{Law of implication} \\
(\neg p \vee \neg q) \vee (\neg p \vee q) & \quad \text{DeMorgan} \\
\neg p \vee \neg q \vee \neg p \vee q & \quad \text{Associative} \\
\neg p \vee \neg q \vee q \vee \neg p & \quad \text{Commutative} \\
\neg p \vee T \vee \neg p & \quad \text{Negation} \\
\neg p \vee T & \quad \text{Domination} \\
T & \quad \text{Domination}
\end{aligned}$$

4. Find the values, if any , of the Boolean variable x that satisfies these equations:

   (a) $x \cdot 1 = 0$ **0**

   (b) $x + x = 0$ **0**

   (c) $x \cdot 1 = x$ **0, 1**

   (d) $x \cdot \bar{x} = 1$ **none**

5. Use truth tables to express the values of these Boolean functions:

   (a) $F(x, y, z) = \overline{xy} + \overline{xz}$

x	y	z	xy	xz	$\overline{xy}$	$\overline{xz}$	$\overline{xy} + \overline{xz}$
0	0	0	0	0	1	1	1
0	0	1	0	0	1	1	1
0	1	0	0	0	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	0	1	1	1
1	0	1	0	1	1	0	1
1	1	0	1	0	0	1	1
1	1	1	1	1	0	0	0

   (b) $F(x, y, z) = \bar{y}(xz + \bar{x}\bar{z})$

x	y	z	$\overline{x}$	$\overline{y}$	$\overline{z}$	xz	$\bar{x}\bar{z}$	$xz + \bar{x}\bar{z}$	$\bar{y}(xz + \bar{x}\bar{z})$
0	0	0	1	1	1	0	1	1	1
0	0	1	1	1	0	0	0	0	0
0	1	0	1	0	1	0	1	1	0
0	1	1	1	0	0	0	0	0	0
1	0	0	0	1	1	0	0	0	0
1	0	1	0	1	0	1	0	1	1
1	1	0	0	0	1	0	0	0	0
1	1	1	0	0	0	1	0	1	0

6. For a Boolean function on each of the following number of inputs:

   - How many rows are in the truth table?
   - How many different Boolean functions are possible?
     - 3 inputs ("a Boolean function of degree 3") 8 rows; 2^8 functions
     - 4 inputs 16 rows; 2^{16} functions
     - 30 inputs 2^{30} rows; $2^{(2^{30})}$ (about 2^{billion}) functions

   In general for n variables there are 2^n rows and $2^{(2^n)}$ possible functions.

7. Half adder

   (a) Write the truth table for a half adder (takes two bits, x and y, and outputs two bits - s (sum) and c (carry):

x	y	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

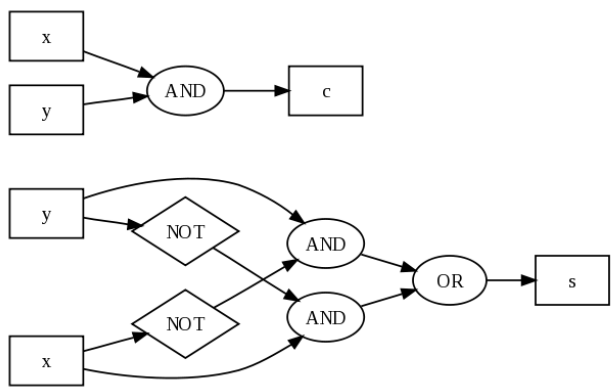
(b) Use the truth table to write the boolean expressions for outputs s and c. (Don't minimize.)

$s = \bar{x}y + x\bar{y}$

$c = xy$

(c) How many gates will you need in a circuit that implements these expressions? **6 gates: 3 AND, 1 OR, 2 NOT**

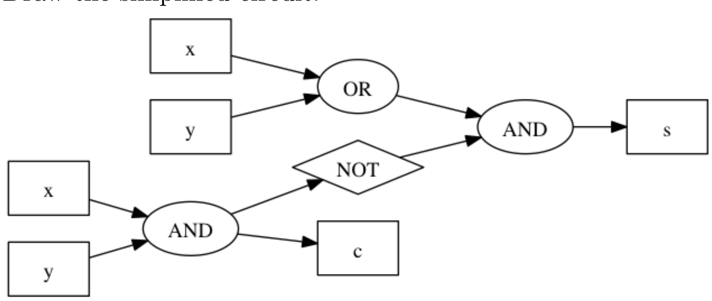
(d) Draw the circuit.



(e) Minimize the expression for s. Now how many gates do you need?

$s = (x + y)\overline{xy}$; **4 gates: 2 AND, 1 OR, 1 NOT** (Notice that we can reuse the xy AND gate.)

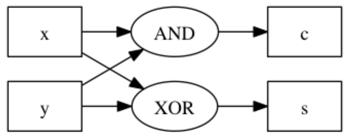
(f) Draw the simplified circuit.



Note: All of the above was done with just AND, OR and NOT gates. If we allow XOR gates, then we can have a much simpler circuit with just 2 gates (1 XOR, 1 AND):

$c = xy$

$s = x \oplus y$



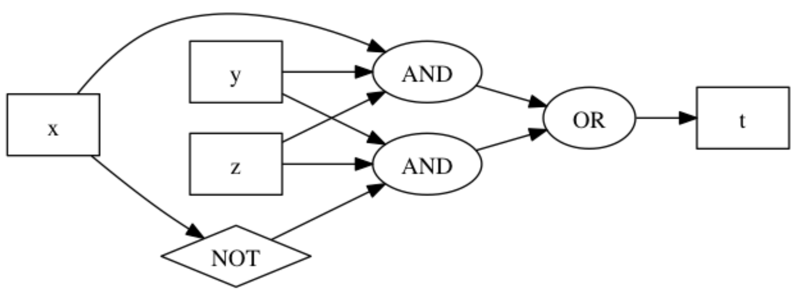
8. Repeat the steps from the above problem (using t as the single output value) for the Boolean function given by the following truth table:

x	y	z	t
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

(a) Use the truth table to write the boolean expression for t:
$t = xyz + \bar{x}yz$

(b) How many gates would you need for this circuit?
4: 2 AND, 1 OR, 1 NOT

(c) Draw the circuit:



(d) Minimize the expression for t. Now how many gates do you need?:
$t = yz$; just 1 ADD gate

(e) Draw the simplified circuit: