University of Washington October 30, 2011
Department of Computer Science and Engineering
CSE 311, Autumn 2011

Sample midterm questions

Instructions:

- Exam will consist of 5 to 8 questions.
- Closed book, closed notes, no cell phones, no calculators.
- Time limit: 50 minutes.
- Answer the problems on the exam paper.
- If you need extra space use the back of a page.
- Lists of equivalences and inference rules for your use are given on the final two pages.

Problem 1:

a) Show that the expression $(p \to q) \to (p \to r)$ is a contingency.

b) Give an expression that is logically equivalent to $(p \to q) \to (p \to r)$ using the logical operators $\neg$, $\vee$, and $\wedge$ (but not $\to$).

Problem 2:
Using the predicates:

$Likes(p, f)$: "Person p likes to eat the food f."
$Serves(r, f)$: "Restaurant r serves the food f."

translate the following statements into logical expressions.

a) Every restaurant serves a food that no one likes.

b) Every restaurant that serves TOFU also serves a food which RANDY does not like.

Problem 3:
Use rules of inference to show that if the premises $\forall x(P(x) \to Q(x))$, $\forall x(Q(x) \to R(x))$, and $\neg R(a)$, where a is in the domain, are true, then the conclusion $\neg P(a)$ is true. (Note: You do not need to give the names for the rules of inference.)

Problem 4:

Prove that if n is even and m is odd, then $(n+1)(m+1)$ is even.

Problem 5:

Prove or disprove:

a) For positive integers x, p, and q, $(x \bmod p) \bmod q = x \bmod pq$.

b) For positive integers x, p, and q, $(x \bmod p) \bmod q = (x \bmod q) \bmod p$.

Problem 6:

a) Find the multiplicative inverse of 2 modulo 9 (in other words, find a solution to the equation $2x \bmod 9 = 1$.)

b) Which integers in $\{1, 2, \ldots, 8\}$ have multiplicative inverses modulo 9?

Problem 7:

Let $T(n)$ be defined by: $T(0) = 1$, $T(n) = 2nT(n-1)$. Prove that for all $n \geq 0$, $T(n) = 2^n n!$.

Problem 8:

Let $x_1, x_2, \ldots, x_n$ be odd integers. Prove by induction that $x_1 x_2 \cdots x_n$ is also an odd integer.

Problem 9:

Determine whether the following compound proposition is a tautology, a contradiction, or a contingency: $((s \vee p) \wedge (s \vee \neg p)) \rightarrow ((p \rightarrow q) \rightarrow r)$.

Problem 10:

Find predicates $P(x)$ and $Q(x)$ such that $\forall x(P(x) \oplus Q(x))$ is true, but $\forall x P(x) \oplus \forall x Q(x)$ is false.

Problem 11:

Show that the following is a tautology: $(((\neg p \vee q) \wedge (p \vee r)) \rightarrow (q \vee r))$.

Problem 12:

Prove that the sum of an odd number and an even number is an odd number.

Problem 13:

Use mathematical induction to show that 3 divides $n^3 - n$ whenever n is a non-negative integer.

Problem 14:

Let the predicates $D(x, y)$ mean "team x defeated team y" and $P(x, y)$ mean "team x has played team y." Give quantified formulas with the following meanings:

a) Every team has lost at least one game.

b) There is a team that has beaten every team it has played.

Problem 15:

Let $P(x, y)$ be the predicate "$x < y$" and let the universe for all variables be the real numbers. Express each of the following statements as predicate logic formulas using P:

a) For every number there is a smaller one.

b) 7 is smaller than any other number.

c) 7 is between a and b. (Don't forget to handle both the possibility that b is smaller than a as well as the possibility that a is smaller than b.)

d) Between any two different numbers there is another number.

e) For any two numbers, if they are different then one is less than the other.

Problem 16:

Let $V(x, y)$ be the predicate "x voted for y", let $M(x, y)$ be the predicate "x received more votes than y", and let the universe for all variables be the set of all people. Express each of the following statements as predicate logic formulas using V and M:

a) Everybody received at least one vote.

b) Jane and John voted for the same person.

c) Ross won the election. (The winner is the person who received the most votes.)

d) Nobody who votes for him/herself can win the election.

e) Everybody can vote for at most one person.

Problem 17:

Prove the following for all natural numbers n by induction, $\displaystyle\sum_{i=0}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$.

Problem 18:

Use Euclid's algorithm to help you solve $11x \equiv 4(\text{mod}27)$ for x.

Problem 19:

Write an expression equivalent to $(p \to q) \to r$ that is:

a) A sum of products

b) A product of sums

Equivalences	
Identity Laws	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$
Domination Laws	$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$
Idempotent Laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative Laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative Laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive Laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Negation Laws	$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$
Double Negation Law	$\neg\neg p \equiv p$
Contrapositive Law	$p \rightarrow q \equiv \neg q \rightarrow \neg p$
Implication Law	$p \rightarrow q \equiv \neg p \vee q$
Quantifier Negation Laws	$\neg \exists x P(x) \equiv \forall x \neg P(x)$ $\neg \forall x P(x) \equiv \exists x \neg P(x)$

Propositional and Predicate Equivalences

Inferences	
Modus Ponens	$\dfrac{p,\ p \rightarrow q}{\therefore q}$
Direct Proof	$\dfrac{p \Rightarrow q}{\therefore p \rightarrow q}$
Elim $\wedge$	$\dfrac{p \wedge q}{\therefore p,\ q}$
Intro $\wedge$	$\dfrac{p,\ q}{\therefore p \wedge q}$
Elim $\vee$	$\dfrac{p \vee q,\ \neg p}{\therefore q}$
Intro $\vee$	$\dfrac{p}{\therefore p \vee q,\ q \vee p}$
Excluded Middle	$\dfrac{}{\therefore p \vee \neg p}$
Elim $\forall$	$\dfrac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$
Intro $\forall$	$\dfrac{\text{Let } a \text{ be anything...} P(a)}{\therefore \forall x P(x)}$
Elim $\exists$	$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for some special } c}$
Intro $\exists$	$\dfrac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$

Propositional and Predicate Inferences