University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2011

November 18, 2011

Homework 8, Due Wednesday, November 23, 2011

Problem 1:

A relation R is called *circular* iff $(c, a) \in R$ whenever $(a, b) \in R$ and $(b, c) \in R$. Prove that for any reflexive relation R, R is circular if and only if R is both symmetric and transitive.

Problem 2:

Let R be the relation on pairs of positive integers, $\mathbb{Z}^+ \times \mathbb{Z}^+$ given by $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Prove that R is reflexive, symmetric and transitive.

Problem 3:

A directed graph is called *acyclic* iff if it does not contain a directed cycle (a non-empty path from a vertex to itself). Show that for every directed acyclic graph G, the transitive-reflexive closure of the relation R represented by G is antisymmetric.

Problem 4:

Let R be the relation defined on $\{1, 2, 3, 4, 5\}$ that consists of $\{(1, 2), (2, 3), (3, 4), (4, 5), (5, 2)\}$.

a) Give the matrix representation of R^2.

b) Give the graph representation of the Transitive-Reflexive closure of R.

Problem 5:

Let R be the relation on $\mathbb{Z} \times \mathbb{Z}$ given by $((a, b), (c, d)) \in R$ if and only if $|a - c| + |b - d| = 1$.
Is the transitive-reflexive closure of R equal to the transitive-reflexive closure of R^2? Justify your answer.

Problem 6:

Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.

a) The set of all binary strings that start with 0 and have even length, or start with 1 and have odd length.

b) The set of all binary strings that have a 1 in every odd-numbered position counting from the start of the string.

Problem 7:

Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.

a) The set of all binary strings that contain at least two 0's and at most one 1.

b) The set of all binary strings that don't contain 110.

Extra Credit 8:

Give a state diagram for a (deterministic) finite state machine that recognizes the set of all binary strings that represent integers that are multiples of 3 when read from left to right. Indicate the start and final states in your diagram and clearly label each state. In addition to the diagram document your design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.