University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2011

October 20, 2011

Homework 4, Due Wednesday, October 26, 2011

Problem 1:
Suppose that $\mathcal{P}(A) = \mathcal{P}(B)$. Show that $A = B$.

Problem 2:
Suppose that $C \neq \emptyset$ and $A \times C = B \times C$. Show that $A = B$. Why do we need the assumption that $C \neq \emptyset$?

Problem 3:
Which, if any, of the following assumptions implies that $A = B$ for all sets A, B, and C? Justify your answers.

(a) $A \cup C = B \cup C$.

(b) $A \cap C = B \cap C$.

(c) Both $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

Problem 4:
Prove that if n is an integer then n^2 mod 5 is either 0, 1, or 4.

Problem 5:
Let a, b be integers and c, n be positive integers.
Prove that if $a \equiv b \pmod{n}$ then $ac \equiv bc \pmod{cn}$.

Problem 6:
Let $E_7^3 = \{x \mid x \equiv 3 \pmod{7}\}$ and $E_{21}^{10} = \{x \mid x \equiv 10 \pmod{21}\}$.
Prove $E_{21}^{10} \subseteq E_7^3$.

Problem 7:
For each $a \in \{1, \ldots, 10\}$ determine the smallest $k \geq 1$ such that $a^k \bmod 11 = 1$.

Problem 8:
How many multiplications are needed to compute 37^{1000} mod 10000 using the fast modular exponentiation algorithm? (You do not need to compute 37^{1000} mod 10000, just determine the number of multiplications and justify your answer.)

Extra Credit 9:

(a) Use a formula for the unsigned decimal expansion of integers to justify the "casting out nines" rule that a positive number is divisible by 9 if and only if repeatedly adding its digits gives 9. (For example, the digit sum for 729 is 18 which has digit sum 9.)

(b) Do the same for the "casting out threes" rule which says that a number is divisible by 3 if and only if the final digit sum is one of 3, 6, or 9.