University of Washington October 16, 2011
Department of Computer Science and Engineering
CSE 311, Autumn 2011

Homework 3, Due Wednesday, October 19, 2011

Problem 1:

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
a) All students in your class can solve quadratic equations.
b) Some student in your class does not want to be rich.

Problem 2:

Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent.

Problem 3:

Let $F(x, y)$ be the statement "x can fool y," where the domain for both x and y constis of all people in the world. Use quantifiers to express each of these statements:
a)Everybody can fool Fred.
b)Everybody can fool somebody.
c)Nancy can fool exactly one person.
d)No one can fool himself or herself.

Problem 4:

Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.
a) $\exists x \forall y(x + y = y)$
b) $\exists x \exists y(((x \leq 0) \wedge (y \leq 0)) \wedge (x - y > 0))$

Problem 5:

Rewrite each of these statements so that the negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
a) $\neg \exists y \exists x P(x, y)$
b) $\neg \exists y(Q(y) \wedge \forall x \neg R(x, y))$
c) $\neg \exists y(\exists x R(x, y) \vee \forall x S(x, y))$

Problem 6:

Show that the premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$ imply the conclusion that $q \rightarrow r$ using the inference rules and equivalences. How many rows would you need if you tried to do this using a truth table?

Problem 7:

Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

Problem 8:

Prove or disprove: $n^2 + 3n + 1$ is always prime for integer $n > 0$.

Extra Credit 9:

Sometimes there are logical inference tasks where possibilities are mutually exclusive and we can express solutions using a simple assignment table rather than separate predicates. Consider the following:

There are 5 differently-colored houses in a row on a street, each occupied by people with different professions, pets, cell phones, and soft drinks of choice. Based on the following conditions, give a table that lists for each house (in order from left to right), the color, profession, pet, soft drink, and cell phone of the occupant in a way consistent with the following:

1. The firefighter lives in the red house.
2. The teacher has a pet snake.
3. Coffee is drunk in the green house.
4. The doctor drinks tea.
5. The green house is immediately to the right of the pink house.
6. The person with the Nokia phone has a pet ferret.
7. An iPhone is used in the yellow house.
8. Milk is drunk in the middle house.
9. The lawyer lives in the left-most house.
10. The Blackberry user lives in the house next to the person with the dog.
11. The iPhone is used in a house next to the house with the pet gerbil.
12. The HTC phone user drinks orange juice.
13. The cab driver uses a Samsung phone.
14. The doctor lives next to the blue house.
15. Someone has a pet lizard.
16. Someone prefers to drink water.

You do **not** need to show your reasoning. NOTE: There was an error in writing this out and #14 should be the lawyer, not the doctor. Answers to either version of the question are OK.