University of Washington
Department of Computer Science and Engineering
CSE 311, Autumn 2011

October 8, 2011

Homework 2, Due Wednesday, October 12, 2011

Problem 1:

Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology by applying a series of equivalences to derive T.

Problem 2:

Show that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not equivalent.

Problem 3:

Find a compound proposition involving the propositional variables p, q, and r, that is true when p and q are true and r is false, but is false otherwise.

Problem 4:

Use truth tables to represent the values of each of these boolean functions:
a) $F(x, y, z) = \overline{x}yz + \overline{(xyz)}$
b) $F(x, y, z) = x(yz + \overline{y}\ \overline{z})$

Problem 5:

Find the sum-of-products expansion of these Boolean functions:
a) $F(x, y, z) = (x + z)y$
b) $F(x, y, z) = x$
c) $F(x, y, z) = x\overline{y}$

Problem 6:

Construct a circuit that computes the product of the two two-bit integers $(x_1x_0)_2$ and $(y_1y_0)_2$. The circuit should have four output bits for the bits in the product.

Problem 7:

Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.
a) $\forall x(R(x) \wedge H(x)$
b) $\exists x(R(x) \to H(x))$

Problem 8:

Let $C(x)$ be the statement "x has a cat", $D(x)$ be the statement "x has a dog", and $F(x)$ be the statement "x has a ferret". Express each of the following statements using quantifiers, logical connectives, and the above statements. The domain consists of all students in the class.
a) All students in the class have a cat, a dog, and a ferret.
b) Some student in the class has a cat and a ferret, but not a dog.
c) For each of the three animals, cats, dogs, and ferrets, there is a student in the class who has one of these animals as a pet.

Extra Credit 9:

Design a boolean circuit that has six inputs, and one output where the output is 1 if exactly three of the inputs are 1, and is 0 otherwise. Your circuit should use as few gates as possible. Provide a brief explanation as to how your circuit works. [I don't know what the best possible result is, I came up with a circuit that uses 13 AND gates, 3 XOR gates, 1 OR gate, and 6 NOT Gates.]