CSE 311 Sample final problems.

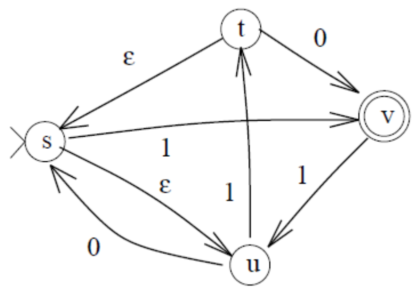
Problem 1:

2. (a) Draw the state diagram of an NFA M that recognizes the language $a^*b(b \cup ab)^*a$. You don't have to use any particular construction method but you should try to avoid unnecessary states.

Problem 2:

The epsilon transition is the same as the lambda transition we used in class.

3. Build a DFA equivalent to the following NFA using the "subset construction." You only need to show states that are reachable from the start state of your DFA (*but* do not attempt to simplify further).



Problem 3:

2. (30 points) Define the language $A = \{w \in \{0,1\}^* \mid$ the number of 0's minus the number of 1's in w is divisible by $3\}$.

   (a) Construct a DFA with only three states that recognizes A.

Problem 4:

Show that the following language cannot be recognized by a DFA:

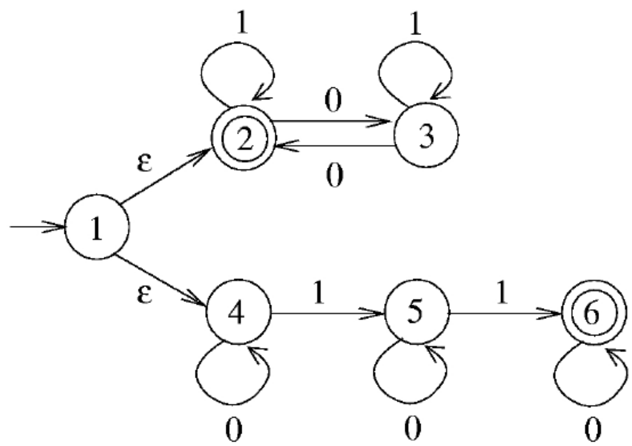
$$\{a^n b a^m b a^{m+n} \mid n, m \geq 1\}$$

Give a CFG for the language

$$\{a^n b a^m b a^{m+n} \mid n, m \geq 1\}$$

Problem 5:

Consider the NFA N = (Q, Σ, δ, q_0, F) with the following state diagram:



a. What states can N be in after reading:
   the string 0? __________ the string 01? __________ the string 0111? _________
b. Does N accept 0111? _____ Why or why not?

Problem 6:

Let L be the set of strings in {a, b}* such that each a, if any, has two b's immediately to its right. Give:

a) A finite automaton accepting L.
b) A regular expression denoting L.
c) A Context free grammar generating L.

Problem 7:

In the land of Garbanzo, the unit of currency is the bean. They only have two coins, one worth 2 beans and the other worth 5 beans.

a. Give a recursive definition of the set of positive integers S such that x is in S if and only if one can make up an amount worth x beans using at most one 5-bean coin and any number of 2-bean coins.
b. Prove by strong induction that every integer >= 4 is in S. (You do NOT need a recursive proof here.)

Problem 8:

Let R be the relation { (1,2), (3,4), (1,3), (2,1) } defined on the set {1,2,3,4,5}.

- Draw the graph of R.
- Draw the graph of the R^2.
- Draw the graph of the reflexive-transitive closure of R.

Problem 9:

For each n>= 0 define T_n>, the 'complete 3-ary tree of height n' as follows:

- T_0 is an undirected graph consisting of a single vertex called the root of T_0.
- For n>= 0, T_{n+1} is an undirected graph consisting of a new vertex of degree 3 joined to the roots of 3 disjoint copies of T_n. The new vertex is called the root of T_{n+1}.

Prove by induction that for each n>= 0, T_n has exactly (3^{n+1}-1)/2 vertices.

Problem 10:

Suppose R_1 and R_2 are transitive-reflexive relations on a set A. Is the relation R_1 UNION R_2 necessarily a transitive-reflexive relation? Justify your answer

Problem 11:

- Every day, starting on day 0, one vampire arrives in Seattle from Transylvania and, starting on the day after its arrival, bites one Seattlite every day. People bitten become vampires themselves and live forever. New vampires also bite one person each day starting the next day after they were bitten. Let V_n be the number of vampires in Seattle on day n. So, for example, $V_0 = 1$, $V_1 = 3$ (one that arrived from Transylvania on day 0, one that he bit on day 1, and another one that arrived from Transylvania on day 1), $V_2 = 7$ and so on. Write a recurrence relation for V_n that is valid for any $n \geq 2$.
- Prove by induction on n that $V_n \leq 3^n$.

Problem 12:

Suppose that for all $n \geq 1$

$$g(n+1) = max_{1 \leq k \leq n}[g(k) + g(n+1-k) + 1]$$

and that $g(1) = 0$. Prove by induction that $g(n) = n - 1$ for all $n \geq 1$.