CSE 311: Foundations of Computing I
Assignment #6
May 10, 2010
due: Monday, May 17, 1:30 p.m.

1. Definition 5 in Section 4.3 defines an extended binary tree. If T_1 and T_2 are both empty, $T_1 \cdot T_2$ is called a *leaf*. For example, the last tree in Figure 3 of Section 4.3 has 2 leaves and the next to last tree in that figure has 3 leaves. The *height* of an extended binary tree is the distance from the root to the farthest leaf. All the trees in Step 3 of Figure 3 have height 2. (Note that the height is considered to be 2 rather than 3: it's the number of edges on the longest root-to-leaf path rather than the number of nodes.) By induction, prove that for any positive integer n, any extended binary tree with n leaves has height at least $\log_2 n$. Be careful of the possibility that your tree has one empty subtree and one nonempty subtree. (Hint: it will be simplest if your induction mirrors the recursive definition given in Definition 5.)

2. Section 8.5, exercise 16.

3. Section 8.5, exercise 40. The answer to part (b) should show that there is a one-to-one correspondence between the equivalence classes of R and the elements of a very familiar mathematical set.

4. Page 584, exercise 10.