CSE 311: Foundations of Computing I
Assignment #3
April 14, 2010
due: Wednesday, April 21, 1:30 p.m.

1. For each of the following functions, state whether or not it is injective, and whether or not it is surjective. Justify your answers.

   (a) $f : \mathbf{N} \to \mathbf{N}$, where $f(n) = n^2$.

   (b) $f : \mathbf{Z} \to \mathbf{N}$, where $f(n) = n^2$.

   (c) $f : \mathbf{R} \to \mathbf{R}$, where $f(n) = 3n + 7$.

   (d) $f : \mathbf{N} \to \mathbf{N}$, where $f(n) = \lceil n/3 \rceil$.

   (e) $f : \mathbf{N} \to \mathbf{N}$, where $f(n) = 3\lceil n/3 \rceil$.

   (f) $f : \mathbf{N} \to \mathbf{N}$, where $f(n) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-1, & \text{if } n \text{ is odd} \end{cases}$.

2. Suppose you graph a function $f : \mathbf{R} \to \mathbf{R}$. The fact that f is a function means that any straight vertical line will intersect the graph of f at exactly one point. What similar statement can you make about the graph of f if f is

   (a) injective?

   (b) surjective?

   (c) bijective?

3. Let $f : \mathbf{N} \to \mathbf{N}$ and $g : \mathbf{N} \to \mathbf{N}$, where $f(x) = x \bmod 28$ and $g(x) = x + 1$. What are each of the functions $f \circ g$ and $g \circ f$? Either prove that these two functions are equal, or give a counterexample proving that they are unequal.

4. Draw the graph of the function $f : \mathbf{R} \to \mathbf{R}$, where $f(x) = \lfloor x/4 \rfloor$. Show the graph for the interval $-12 \leq x \leq 12$.

5. Section 3.4, exercise 4. Give a careful proof.

6. Section 3.5, exercise 6. Justify your answer. The function $n!$ is defined on page 145. (Hint: Think about the unique factorization of 100! into primes. What about this factorization determines the number of zeros at the end of the decimal representation of 100! ?)