



# Elementary statistics

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# A dice-rolling game

- Two players each roll a die
- The higher roll wins
  - Goal: roll as high as you can!
- Repeat the game 6 times

# Hypotheses regarding the outcome

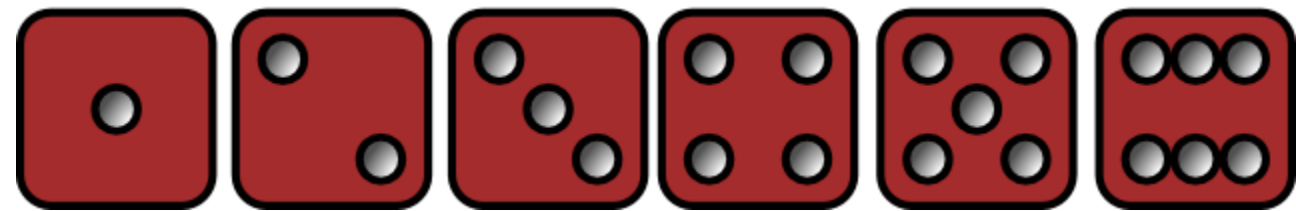
- Luck
- Fraud
  - loaded die
  - inaccurate reporting
- How likely is luck?
- How do we decide?



# Questions that statistics can answer

- I am flipping a coin. Is it fair?  
**How confident am I in my answer?**
- I have two bags of beans, each containing some black and some white beans. I have a handful of beans. Which bag did the handful come from?
- I have a handful of beans, and a single bag. Did the handful come from that bag?
- Does this drug improve patient outcomes?
- Which website design yields greater revenue?
- Which baseball player should my team draft?
- What premium should an insurer charge?
- Which chemical process leads to the best-tasting beer?

# What can happen when you roll a die?



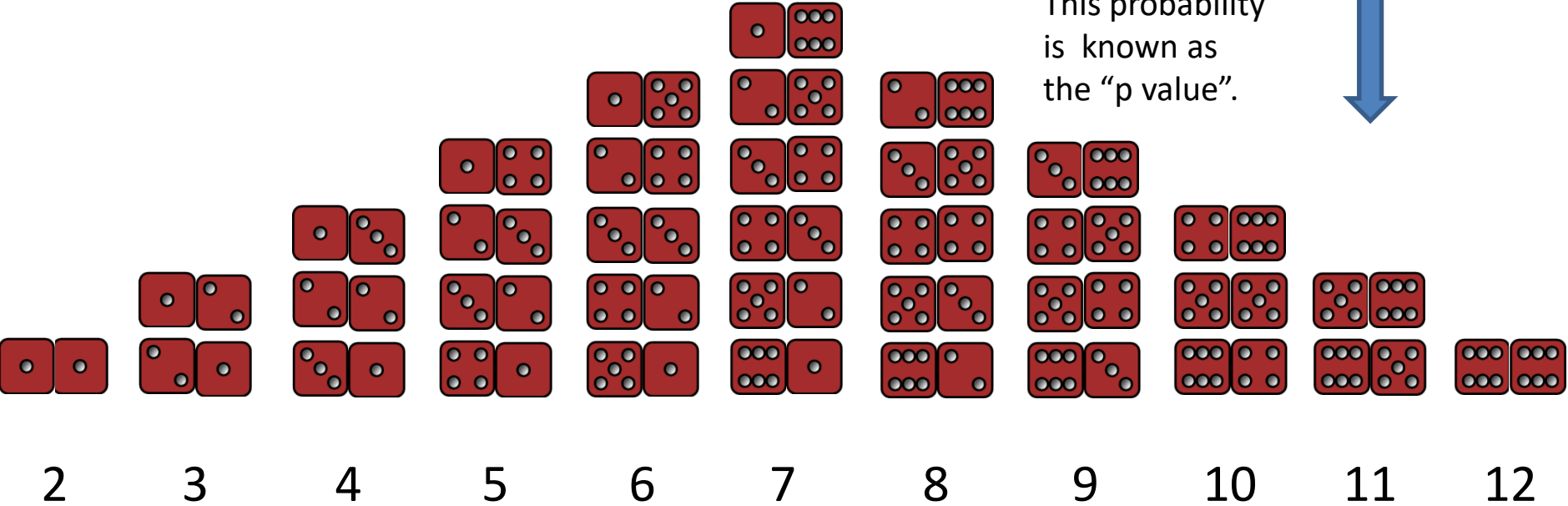
What is the likelihood of each?



# What can happen when you roll two dice?

How likely are you to roll *11 or higher*?

This probability is known as the “p value”.



# How to compute p values

- Via a statistical formula
  - Requires you to make assumptions and know which formula to use
- Computationally (simulation)
  - Run many experiments
  - Count the fraction with a better result
    - Requires a metric/measurement for “better”
  - Requires you to be able to run the experiments
  - We will use this approach exclusively

# Analogy between hypothesis testing and mathematical proofs

“The underlying logic [of hypothesis testing] is similar to a proof by contradiction. To prove a mathematical statement,  $A$ , you assume temporarily that  $A$  is false. If that assumption leads to a contradiction, you conclude that  $A$  must actually be true.”

From the book *Think Statistics* by Allen Downey



# Summary of statistical methodology

1. Decide on a metric (bigger value = better)
2. Observe what you see in the real world
3. Hypothesize that what you saw is normal/typical  
This is the “null hypothesis”

4. Simulate the real world many times

5. How different is what you observed from the simulations?

What percent of the simulation values are the real world values bigger than?

6. If the percentage is 95% or more, reject the null hypothesis

# Null Hypothesis

Null Hypothesis: The common wisdom, “nothing unusual is happening here”

Examples:

- Ruth was using a fair die
- The accused is innocent
- This new drug does NOT cure disease
- The Iranian election results are accurate

# Interpreting p values

p value of 5% or less = statistically significant

- This is a *convention*; there is nothing magical about 5%

Two types of errors may occur in statistical tests:

- **false positive** (or **false alarm** or Type I error): no real effect, but report an effect (through good/bad luck or coincidence)
  - If no real effect, a false positive occurs about 1 time in 20
- **false negative** (or **miss** or Type II error): real effect, but report no effect (through good/bad luck or coincidence)

The *larger* the sample, the *less the likelihood* of a false positive or negative

# Errors

Type 1: False Positive (false alarm)

Type 2: False negative (miss)

Examples:

- Ruth was using a fair die
  - Type 1: Die is actually fair, accuse me of lying!
  - Type 2: Die is actually biased, you don't notice
- The accused is innocent
- This new drug does NOT cure disease
- The Iranian election results are accurate

# Error Examples

Type 1: False Positive (false alarm)

Type 2: False negative (miss)

Examples:

- Ruth was using a fair die
  - Type 1: Die is actually fair, accuse me of lying!
  - Type 2: Die is actually biased, you don't notice
- The accused is innocent
  - Type 1:
  - Type 2:
- This new drug does NOT cure disease
  - Type 1:
  - Type 2:
- The Iranian election results are fair/accurate
  - Type 1:
  - Type 2:

# Answer: Error Examples

Type 1: False Positive (false alarm)

Type 2: False negative (miss)

Examples:

- Ruth was using a fair die
  - Type 1: Die is actually fair, accuse me of lying!
  - Type 2: Die is actually biased, you don't notice
- The accused is innocent
  - Type 1: Actually innocent, court finds guilty
  - Type 2: Actually guilty, court sets them free
- This new drug does NOT cure disease
  - Type 1: Drug actually does nothing, study claims it does
  - Type 2: Drug actually does help, study claims it does not
- The Iranian election results are fair/accurate
  - Type 1: Results are actually fair, we claim they are fraudulent
  - Type 2: Results are actually fraudulent, we claim they are fair

# A false positive

JELLY BEANS CAUSE ACNE!

SCIENTISTS!  
INVESTIGATE!

BUT WE'RE  
PLAYING  
MINECRAFT!  
...FINE.

WE FOUND NO  
LINK BETWEEN  
JELLY BEANS AND  
ACNE ( $P > 0.05$ ).

THAT SETTLES THAT.

I HEAR IT'S ONLY  
A CERTAIN COLOR  
THAT CAUSES IT.

SCIENTISTS!

BUT  
MINECRAFT!

WE FOUND NO  
LINK BETWEEN  
PURPLE JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
BROWN JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
PINK JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
BLUE JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
TEAL JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
SALMON JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
RED JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
TURQUOISE JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
MAGENTA JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO  
LINK BETWEEN  
YELLOW JELLY  
BEANS AND ACNE  
( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN PURPLE JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN BROWN JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN PINK JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN BLUE JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN TEAL JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN SALMON JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN RED JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN TURQUOISE JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN MAGENTA JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN YELLOW JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN GREY JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN TAN JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN CYAN JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND A LINK BETWEEN GREEN JELLY BEANS AND ACNE ( $P < 0.05$ ).

WE FOUND NO LINK BETWEEN MAUVE JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN BEIGE JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN LILAC JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN BLACK JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN PEACH JELLY BEANS AND ACNE ( $P > 0.05$ ).

WE FOUND NO LINK BETWEEN ORANGE JELLY BEANS AND ACNE ( $P > 0.05$ ).


**News**

**GREEN JELLY BEANS LINKED TO ACNE!**

**95% CONFIDENCE**

**ONLY 5% CHANCE OF COINCIDENCE!**

**SCIENTISTS...**





# A common error

1. Observe what you see in the real world
2. Decide on a metric (bigger value = better)

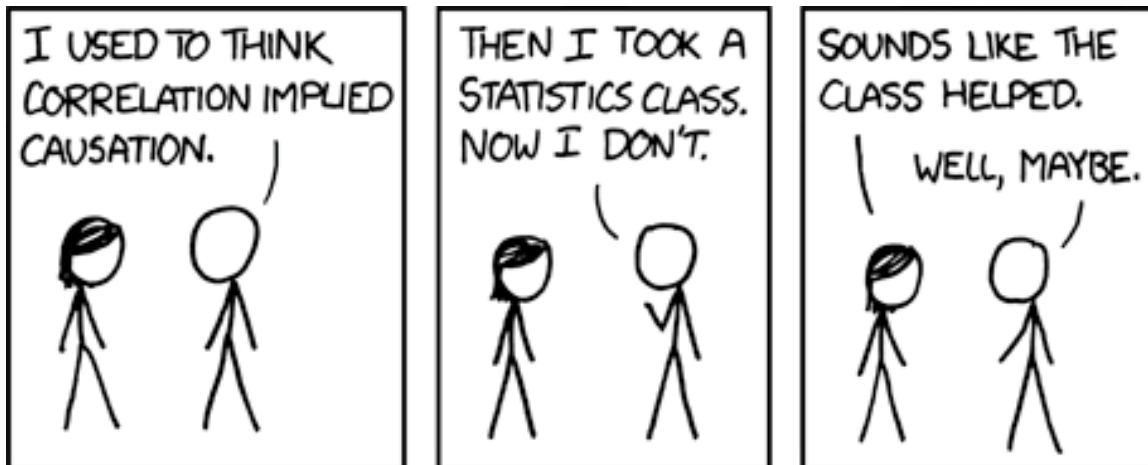
This is *backwards*

For any observation, there is something unique about it.

Example: Roll dice, then be amazed because what are the odds you would get exactly that combination of rolls?

# Correlation $\neq$ causation

Ice cream sales and rate of drowning deaths are correlated



**Statistical significance  
≠ practical importance**