Building Java Programs

Binary Trees

reading: 17.1 – 17.3
THE JOB

PLANT THE DECISION TREE THERE.

LANDSCAPING AT THE INDUSTRIAL PARK.
Trees

- **tree**: A directed, acyclic structure of linked nodes.
  - *directed*: Has one-way links between nodes.
  - *acyclic*: No path wraps back around to the same node twice.

- **binary tree**: One where each node has at most two children.

- **Recursive definition**: A tree is either:
  - empty (*null*), or
  - a *root* node that contains:
    - data,
    - a *left* subtree, and
    - a *right* subtree.
      - *(The left and/or right subtree could be empty.)*
Trees in computer science

- TreeMap and TreeSet implementations
- folders/files on a computer
- family genealogy; organizational charts
- AI: decision trees
- compilers: parse tree
  - \( a = (b + c) \times d \); 
- cell phone T9
Terminology

• **node**: an object containing a data value and left/right children
  • **root**: topmost node of a tree
  • **leaf**: a node that has no children
  • **branch**: any internal node; neither the root nor a leaf
  • **parent**: a node that refers to this one
  • **child**: a node that this node refers to
  • **sibling**: a node with a common parent

• **subtree**: the smaller tree of nodes on the left or right of the current node
• **height**: length of the longest path from the root to any node
• **level** or **depth**: length of the path from a root to a given node
Recursive data structure

- Recursive definition: A tree is either:
  - empty (null), or
  - a root node that contains:
    - data,
    - a left tree, and
    - a right tree
A tree node for integers

- A basic **tree node object** stores data, refers to left/right
- Multiple nodes can be linked together into a larger tree
// An IntTreeNode object is one node in a binary tree of ints.
public class IntTreeNode {
    public int data;       // data stored at this node
    public IntTreeNode left;  // reference to left subtree
    public IntTreeNode right; // reference to right subtree

    // Constructs a leaf node with the given data.
    public IntTreeNode(int data) {
        this(data, null, null);
    }

    // Constructs a branch node with the given data and links.
    public IntTreeNode(int data, IntTreeNode left,
                        IntTreeNode right) {
        this.data = data;
        this.left = left;
        this.right = right;
    }
}

<table>
<thead>
<tr>
<th>left</th>
<th>data</th>
<th>right</th>
</tr>
</thead>
</table>
// An IntTree object represents an entire binary tree of ints.
public class IntTree {
    private IntTreeNode overallRoot;  // null for an empty tree

    methods
}

- Client code talks to the IntTree, not to the node objects inside it.

- Methods of the IntTree create and manipulate the nodes, their data and links between them.
**IntTree constructors**

- For now, assume we have the following constructors:

  ```java
  public IntTree(IntTreeNode overallRoot)
  public IntTree(int height)
  ```

- The 2nd constructor will create a tree and fill it with nodes with random data values from 1-100 until it is full at the given height.

```java
IntTree tree = new IntTree(3);
```
Traversals

- **traversal**: An examination of the elements of a tree.
  - A pattern used in many tree algorithms and methods

- Common orderings for traversals:
  - **pre-order**: process root node, then its left/right subtrees
  - **in-order**: process left subtree, then root node, then right
  - **post-order**: process left/right subtrees, then root node
Traversals example

- **pre-order:** 17 41 29 6 9 81 40
- **in-order:** 29 41 6 17 81 9 40
- **post-order:** 29 6 41 81 40 9 17
Traversals trick

To quickly generate a traversal:
- Trace a path around the tree.
- As you pass a node on the proper side, process it.
  - pre-order: left side
  - in-order: bottom
  - post-order: right side

pre-order: 17 41 29 6 9 81 40
in-order: 29 41 6 17 81 9 40
post-order: 29 6 41 81 40 9 17
Exercise

• Give pre-, in-, and post-order traversals for the following tree:

- pre: 42 15 27 48 9 86 12 5 3 39
- in: 15 48 27 42 86 5 12 9 3 39
- post: 48 27 15 5 12 86 39 3 42
Exercise

- Add a method `print` to the `IntTree` class that prints the elements of the tree, separated by spaces.
  - A node's left subtree should be printed before it, and its right subtree should be printed after it.

- Example: `tree.print();`

```
29 41 6 17 81 9 40
```
// An IntTree object represents an entire binary tree of ints.
public class IntTree {
    private IntTreeNode overallRoot; // null for an empty tree
    ...

    public void print() {
        print(overallRoot);
        System.out.println(); // end the line of output
    }

    private void print(IntTreeNode root) {
        // (base case is implicitly to do nothing on null)
        if (root != null) {
            // recursive case: print left, center, right
            print(overallRoot.left);
            System.out.print(overallRoot.data + " ");
            print(overallRoot.right);
        }
    }
}
Template for tree methods

```java
class IntTree {
    private IntTreeNode overallRoot;
    ...

    public type name(parameters) {
        name(overallRoot, parameters);
    }

    private type name(IntTreeNode root, parameters) {
        ...
    }
}
```

- Tree methods are often implemented recursively
  - with a public/private pair
  - the private version accepts the root node to process
Exercise

- Add a method \texttt{contains} to the \texttt{IntTree} class that searches the tree for a given integer, returning \texttt{true} if it is found.

  - If an \texttt{IntTree} variable \texttt{tree} referred to the tree below, the following calls would have these results:

    \begin{itemize}
    \item \texttt{tree.contains(87)} \rightarrow \texttt{true}
    \item \texttt{tree.contains(60)} \rightarrow \texttt{true}
    \item \texttt{tree.contains(63)} \rightarrow \texttt{false}
    \item \texttt{tree.contains(42)} \rightarrow \texttt{false}
    \end{itemize}
// Returns whether this tree contains the given integer.
public boolean contains(int value) {
    return contains(overallRoot, value);
}

private boolean contains(IntTreeNode node, int value) {
    if (node == null) {
        return false;  // base case: not found here
    } else if (node.data == value) {
        return true;  // base case: found here
    } else {
        // recursive case: search left/right subtrees
        return contains(node.left, value) ||
                contains(node.right, value);
    }
}
Exercise

• Add a method named `printSideways` to the `IntTree` class that prints the tree in a sideways indented format, with right nodes above roots above left nodes, with each level 4 spaces more indented than the one above it.

  – Example: Output from the tree below:

```
  9
 / \
14 19
 \
 11
 \
  6
```

```
 overall root
  \
  9
    \
   14
     \
    11
     \
    6
```
public void printSideways(IntTreeNode root, String indent) {
    if (root != null) {
        printSideways(root.right, indent + "    ");
        System.out.println(indent + root.data);
        printSideways(root.left, indent + "    ");
    }
}