Recursive Backtracking

Definition (Recursive Backtracking)
Recursive Backtracking is an attempt to find solution(s) by building up partial solutions and abandoning them if they don’t work.

Recursive Backtracking Strategy
- If we found a solution, stop looking (e.g. return)
- Otherwise for each possible choice $c$
  - Make the choice $c$
  - Recursively continue to make choices
  - Un-make the choice $c$ (if we got back here, it means we need to continue looking)

Words & Permutations

All Words
Find all length $n$ strings made up of $a$’s, $b$’s, and $c$’s.

To do this, we build up partial solutions as follows:
- The only length 0 string is ""; so, we’re done.
- Otherwise, the three choices are $a$, $b$, and $c$:
  - Make the choice letter
  - Find all solutions with one fewer letter recursively.
  - Unmake the choice (to continue looking).

All Words Solution

```java
private static void words(int length) {
    String[] choices = {"a", "b", "c", "d"};
    if (length == 0) {
        print();
    } else {
        for (String choice : choices) {
            choose(choice);
            words(length - 1);
            unchoose();
        }
    }
}
```
private static void words(String acc, int length) {
    if (length == 0) {
        print();
    } else {
        for (String choice : choices) {
            acc += choice;
            words(acc, length - 1);
            acc = acc.substring(0, acc.length() - 1);
        }
    }
}

public static boolean canSolveMaze(int x, int y) {
    if (isGoal(x, y)) {
        return true;
    } else if (!isOOB() && isPassage(panel)) {
        p.makeVisited(panel);
        panel.sleep(120);
        if (solveMaze(p.getLeft()) ||
            solveMaze(p.getRight()) ||
            solveMaze(p.getAbove()) ||
            solveMaze(p.getBelow())) {
            return true;
        }
        panel.sleep(200);
        p.makeDeadEnd(panel); // Undo the choice
    } return false;
}

public boolean solveMaze(Point p) {
    // We found a path to the goal!
    p.makeVisited(panel);
    return true;
}

Recursion Reminder

Solving Recursion Problems
- Figure out what the pieces of the problem are.
- What is the base case? (the smallest possible piece of the problem)
- Solve one piece of the problem and recurse on the rest.

paintbucket Review
- A piece of the problem is one surrounding set of squares
- The base case is we hit a non-white cell
- To solve one piece of the problem, we color the cell and go left, right, up, and down

Recursive Backtracking Tips!

- The most important part is figuring out what the choices are.
- It can help to draw out a tree of choices
- Make sure to undo your choices after the recursive call.
- You will still always have a base case.

Solving a Maze

Solving a maze is a lot like paintbucket. What is the difference?
Instead of filling everything in, we want to stop at dead ends!
If you were in a maze, how would you solve it?

- Try a direction.
- Every time you go in a direction, draw an X on the ground.
- If you hit a dead end, go back until you can go in another direction.

This is recursive backtracking!

1 public boolean canSolveMaze(int x, int y) {
2     if (isGoal(x, y)) {
3         return true;
4     } else if (inBounds(x, y) && isPassage(x, y)) {
5         return solveMaze(x + 1, y) ||
6             solveMaze(x - 1, y) ||
7             solveMaze(x, y + 1) ||
8             solveMaze(x, y - 1);
9     }
10 }
11

1 public boolean solveMaze(Point p) {
2     if (p.isGoal()) {
3         p.makeVisited(panel);
4         return true;
5     } else if (p.isOOB() && p.isPassage(panel)) {
6         p.makeVisited(panel); // Choose this point
7         panel.sleep(120);
8         if (solveMaze(p.getLeft()) ||
9             solveMaze(p.getRight()) ||
10             solveMaze(p.getAbove()) ||
11             solveMaze(p.getBelow())) {
12             return true;
13         }
14         panel.sleep(200);
15         p.makeDeadEnd(panel); // Undo the choice
16     }
17     return false;
18 }
19

Accumulators

4

1 private static void words(String acc, int length) {
2     String[] choices = {"a", "b", "c", "d"};
3     // The empty string is the only word of length 0
4     if (length == 0) {
5         print();
6     } else {
7         for (String choice : choices) {
8             acc += choice;
9             words(acc, length - 1);
10             acc = acc.substring(0, acc.length() - 1);
11         }
12     }
13 }
14 }
Recursive Backtracking

Definition (Recursive Backtracking)

Recursive Backtracking is an attempt to find solution(s) by building up partial solutions and abandoning them if they don’t work.

Recursive Backtracking Strategy

- If we found a solution, stop looking (e.g. return)
- Otherwise for each possible choice c...
  - Make the choice c
  - Recursively continue to make choices
  - Un-make the choice c (if we got back here, it means we need to continue looking)

NQueens Problem

The NQueens problem is the challenge to place n queens on a chess board so that none of them are attacking each other.

We will begin by solving this problem using for loops, and then we will solve it much more elegantly using recursive backtracking.

Implementing a Tiny Piece of Google

When you enter a query with no spaces like thisisasentence into Google:

```
thisisasentence
```

It fixes it into this is a sentence using recursive backtracking.

Sentence Splitting

Given an input string, sentence, containing no spaces, write a method:

```
public static String splitSentence(String sentence)
```

that returns sentence split up into words.
Sentence Splitting

Given an input string, sentence, containing no spaces, write a method:

```
public static String splitSentence(String sentence)
```

that returns `sentence` split up into words.

To do recursive backtracking, we need to answer these questions:
- What are the choices we're making incrementally? ...which character to split at
- How do we "undo" a choice? ...re-combine a string by the char we split at
- What are the base case(s)? ...our left choice isn't a word and our right choice IS a word

It helps to answer these questions for a particular input. So, pretend we're working with:

```
thisisasentence
```

One More Important Choice

When doing recursive backtracking, we need to differentiate between:
- finding a result
- failing to find a result (e.g., backtracking)

Generally, we do this by treating `null` as a failure. For example:
- On the input, "thisisasentence", none of the recursive calls should return "thisis", because it isn't a word.
- If we get down to an empty string, that would indicate a failure; so, we'd return `null`

Sentence Splitter Solution

```java
1 public String splitSentence(String sentence) {
2     // The entire sentence is a dictionary word!
3     if (words.contains(sentence)) {
4         return sentence;
5     }
6     // Try splitting at every character until we find one that works...
7     for (int i = sentence.length() - 1; i > 0; i--){
8         String left = sentence.substring(0, i);
9         String right = sentence.substring(i, sentence.length());
10        // If the left isn't a word, don't bother recursing.
11        // If it is, split the remainder of the sentence recursively.
12        if (words.contains(left)) {
13            right = splitSentence(right);
14            // Since the left was a word, if the right is also an answer,
15            // then we found an answer to the whole thing!
16            if (right != null) {
17                return left + " " + right;
18            }
19        }
20        // Undo our choice by going back to sentence
21    }
22    return null;
23 }
```