CSE 143

Computer Programming II

Recursive Backtracking

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Outline

Definition (Recursive Backtracking)

Recursive Backtracking is an attempt to find solution(s) by building up partial solutions and abandoning them if they don't work.

Recursive Backtracking Strategy

- If we found a solution, stop looking (e.g. return)
- Otherwise for each possible choice c...
  - Make the choice c
  - Recursively continue to make choices
  - Un-make the choice c (if we got back here, it means we need to continue looking)

Client vs. Implementor, again

You may have noticed that many of the class examples I've been showing involve me using a class that I've already written. There are several reasons for this:

- Learning to read and use an API is a really important programming skill
- Switching between the client and implementor views is an important goal of this course
- The code I write is usually easy, but really tedious (so, it would be a waste of time to write in class)

Take-Away

Every time I print out an API for you, you should try to understand it from the comments. This will help you on the homework, on exams, and in any future programming endeavors.

Today's API is `BooleanExpression`.

What is a `BooleanExpression`?

The `BooleanExpression` class allows us to represent the conditions we write in `if` statements. For instance, to represent the following:

```
1 if (!(queue.size() > 0) && queue.peek() > 5) {
2     ...
3 }
```

We would do

```
new BooleanExpression("(!a && b)");
```

Notice that we use single letter variable names instead of `queue.size() > 0`. This is a simplification for implementation.

Evaluating BooleanExpressions

Remember when we took $(1+2)*3$ and evaluated it to 9 recursively?
We can do a similar thing for BooleanExpressions:
Consider the BooleanExpression from above:

```
"(!a && b)"
```

Suppose we know the following:

- a is true.
- b is false.

What does this expression evaluate to?

`(!a && b)` $\longrightarrow$ `(!true && false)` $\longrightarrow$ `(false && false)` $\longrightarrow$ `false`

Suppose we wanted to write a method:

```
public static boolean evaluate(BooleanExpression e, ???  assn)
```

where assn represents the truth values of the variables.

What type would assn be? It's a **mapping** from variables to truth values.

Okay, so, we have:

```
public static boolean evaluate(BooleanExpression e,
        Map<String, Boolean> assignments)
```

Consider the following case:

`evaluate` return value?

- `e` is `a && b`
- `assignments` map is `{a=true}`.

What should `evaluate` return?

We can't answer the question. What seems like a good idea? **`null`**.

So, we change the return type to `Boolean`.

Who Writes `evaluate`?

- The implementor of `BooleanExpression`
  ...if so, it should be inside the `BooleanExpression` class
- The client of `BooleanExpression`
  ...if so, it should be outside the `BooleanExpression` class

The implementor of `BooleanExpression` should write the method, because then all the clients can use it.

That pesky `static`...

- If the implementor writes `evaluate`, then the method signature is:

```
public Boolean evaluate(Map<String, Boolean> assn)
```

- If the client writes `evaluate`, then the method signature is:

```
public static Boolean evaluate(
   BooleanExpression e,
   Map<String, Boolean> assn
)
```

`canBeTrue`

Write a method

```
public static void canBeTrue(BooleanExpression b)
```

that returns true if it is possible for the input to to **evaluate to true** and false otherwise.

Some examples:

- `a && b` $\longrightarrow$ if we have `{a=true, b=true}`, then it is true.
- `a && !a` $\longrightarrow$ no matter what `a` is, this will always be false.

To do recursive backtracking, we need to answer these questions:

- What are the choices we're making incrementally?
- How do we "undo" a choice?
- What are the base case(s)?

Finally, Back To Recursive Backtracking...

`canBeTrue`

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To do recursive backtracking, we need to answer these questions:

- What are the choices we're making incrementally?
  ... assignments of each variable to true/false
- How do we "undo" a choice?
  ... remove the assignment from the map
- What are the base case(s)?
  ... the assignment must be true/false

Uh Oh. . . How can we try assignments?

We don't have a way of passing assignments through to the function. How can we fix this?

`public/private` **pair!**

Public/Private Recursive Pair

```
public static void canBeTrue(BooleanExpression b)

private static void canBeTrue(
   BooleanExpression b,
   Map<String, Boolean> m
)
```

canBeTrue Solution


```
1  public static void canBeTrue(BooleanExpression b) {
2     Map<String, Boolean> assignmentMap = new TreeMap<>();
3     canBeTrue(b, assignmentMap);
4  }
5
6  private static void canBeTrue(BooleanExpression b, Map<String, Boolean> m) {
7     Set<String> variables = b.getVariables();
8     if (variables.size() == m.keySet().size() && b.evaluate(m)) {
9        System.out.println(m);
10    }
11
12    for (String variable : variables) { // Try to assign any
13       if (!m.keySet().contains(variable)) { // variable we haven't
14          boolean[] choices = {true, false}; // already assigned.
15          for (boolean assignment : choices) {
16             m.put(variable, assignment);
17             canBeTrue(b, m);
18             m.remove(variable);        // Otherwise, backtrack
19          }
20    }
21 }
```

BTW, Why does this problem matter?

Solving `canBeTrue` quickly is the **most important** open problem in Computer Science.

If you solve this problem in $\mathcal{O}(n^k)$ time for **any** k, the following happen:

- You get **one million** dollars.
- You get a PhD.
- You become the most famous Computer Scientist, pretty much ever
- You break all banks, credit cards, website encryption, etc.