1. Recursive Tracing, 15 points. Consider the following method:

```java
public void mystery(int n) {
    System.out.print(n % 10);
    if (n >= 3) {
        mystery(n / 2);
    }
    if (n % 2 == 0) {
        System.out.print("+");
    } else {
        System.out.print("-");
    }
}
```

For each call below, indicate what output is produced:

<table>
<thead>
<tr>
<th>Method Call</th>
<th>Output Produced</th>
</tr>
</thead>
<tbody>
<tr>
<td>mystery(2);</td>
<td>___________________</td>
</tr>
<tr>
<td>mystery(5);</td>
<td>___________________</td>
</tr>
<tr>
<td>mystery(7);</td>
<td>___________________</td>
</tr>
<tr>
<td>mystery(18);</td>
<td>___________________</td>
</tr>
<tr>
<td>mystery(21);</td>
<td>___________________</td>
</tr>
</tbody>
</table>

2. Recursive Programming, 15 points. Write a recursive method called undouble that takes a string as a parameter and that returns a new string obtained by replacing every pair of repeated adjacent letters with one of that letter. For example, the String "bookkeeper" has three repeated adjacent letters ("oo", "kk", and "ee"), so undouble("bookkeeper") should return the string "bokeper". Below are more sample calls:

<table>
<thead>
<tr>
<th>Method Call</th>
<th>Value Returned</th>
<th>Method Call</th>
<th>Value Returned</th>
</tr>
</thead>
<tbody>
<tr>
<td>undouble(&quot;odegaard&quot;)</td>
<td>&quot;odegaard&quot;</td>
<td>undouble(&quot;oops&quot;)</td>
<td>&quot;ops&quot;</td>
</tr>
<tr>
<td>undouble(&quot;baz&quot;)</td>
<td>&quot;baz&quot;</td>
<td>undouble(&quot;foobard&quot;)</td>
<td>&quot;fobar&quot;</td>
</tr>
<tr>
<td>undouble(&quot;mississippi&quot;)</td>
<td>&quot;misisipi&quot;</td>
<td>undouble(&quot;apple&quot;)</td>
<td>&quot;aple&quot;</td>
</tr>
<tr>
<td>undouble(&quot;carry&quot;)</td>
<td>&quot;cary&quot;</td>
<td>undouble(&quot;berry&quot;)</td>
<td>&quot;bery&quot;</td>
</tr>
<tr>
<td>undouble(&quot;juggle&quot;)</td>
<td>&quot;juggle&quot;</td>
<td>undouble(&quot;theses&quot;)</td>
<td>&quot;theses&quot;</td>
</tr>
<tr>
<td>undouble(&quot;little&quot;)</td>
<td>&quot;litle&quot;</td>
<td>undouble(&quot;&quot;)</td>
<td>&quot;&quot;</td>
</tr>
</tbody>
</table>

You may assume that the string is composed entirely of lowercase letters, as in the examples above, and that no letter appears more than two times in a row. Notice that the method might be passed an empty string, in which case it returns an empty string. You are not allowed to construct any structured objects to solve this problem other than strings (no array, ArrayList, StringBuilder, Scanner, etc) and you may not use a while loop, for loop or do/while loop to solve this problem; you must use recursion. You may use only the string methods included on the cheat sheet.
3. Collections Programming, 15 points. Write a method called byAge that takes three parameters: 1) a map where each key is a person's name (a String) and the associated value is that person's age (an integer); 2) an integer for a minimum age; and 3) an integer for a max age. Your method should return a new map with information about people with ages between the min and max, inclusive.

In your result map, each key is an integer age, and the value for that key is a String with the names of all people at that age, separated by "and" if there is more than one person of that age. Include only ages between the min and max inclusive, where there is at least one person of that age in the original map. If the map passed in is empty, or if there are no people in the map between the min/max ages, return an empty map.

For example, if a map called ages stores the following key=value pairs:

{Paul=28, David=20, Janette=18, Marty=35, Stuart=98, Jessica=35, Helene=40, Allison=18, Sara=15, Grace=25, Zack=20, Galen=15, Erik=20, Tyler=6, Benson=48}

The call of byAge(ages, 16, 25) should return the following map (the contents can be in any order):

{18=Janette and Allison, 20=David and Zack and Erik, 25=Grace}

For the same map, the call of byAge(ages, 20, 40) should return the following map:

{20=David and Zack and Erik, 25=Grace, 28=Paul, 35=Marty and Jessica, 40=Helene}

For full credit, obey the following restrictions in your solution. A solution that disobeys them can get partial credit.

* You will need to construct a map to store your results, but you may not use any other structures (arrays, lists, etc.) as auxiliary storage. (You can have as many simple variables as you like.)
* You should not modify the contents of the map passed to your method.
* Your solution should run in no worse than $O(N \log N)$ time, where $N$ is the number of pairs in the map.
4. Linked Lists, 15 points. Fill in the "code" column in the following table providing a solution that will turn the "before" picture into the "after" picture by modifying links between the nodes shown. You are not allowed to change any existing node's data field value and you are not allowed to construct any new nodes, but you are allowed to declare and use variables of type ListNode (often called "temp" variables). You are limited to at most two variables of type ListNode for each of the four subproblems below.

You are writing code for the ListNode class discussed in lecture:

```java
public class ListNode {
    public int data;       // data stored in this node
    public ListNode next;  // link to next node in the list

    <constructors>
}
```

As in the lecture examples, all lists are terminated by null and the variables p and q have the value null when they do not point to anything.

<table>
<thead>
<tr>
<th>before</th>
<th>after</th>
<th>code</th>
</tr>
</thead>
<tbody>
<tr>
<td>q-&gt;[3]</td>
<td>q</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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</tbody>
</table>
5. Array Programming, 10 points. Write a method called removeMax that removes the largest value from a list of integers. For example, if a variable called list stores this sequence of values:

```
[3, 1, 5, 7, 3, 19, 42, 8, 23, 7, 42, 2, -8, 9, 105, -3]
```

and the following call is made:
```
list.removeMax();
```
Then the largest value (105) is removed, leaving this list:
```
[3, 1, 5, 7, 3, 19, 42, 8, 23, 7, 42, 2, -8, 9, -3]
```

If the maximum occurs more than once, the method should remove the first occurrence. For example, if the same call is made again, the list becomes:
```
[3, 1, 5, 7, 3, 19, 42, 8, 23, 7, 42, 2, -8, 9, -3]
```
You are writing a method for the ArrayIntList class discussed in lecture:
```
public class ArrayIntList {
    private int[] elementData; // list of integers
    private int size;          // current # of elements in the list

    <methods>
}
```
The method should throw an IllegalStateException if the list is empty because then there would be no maximum value to remove.

You may not call any other methods of the ArrayIntList class to solve this problem, you are not allowed to define any auxiliary data structures (no array, String, ArrayList, etc), and your solution must run in O(n) time where n is the length of the original list.
6. Stacks/Queues, 25 points. Write a method called makePalindrome that takes a stack of integers as a parameter and that removes pairs of values in mirror positions that don't match, resulting in a sequence of values that is a palindrome. A palindrome is a sequence of values that is the same in backwards order as it is in forwards order. For example, suppose that a stack s stores the following values:

```
   bottom [8, 12, 5, 3, 3, 6, 12, 7] top
^ ^ ^ ^ ^ ^ ^ ^
| | | +----+ | | |
| | +----------+ | |
| +----------------+ |
+-------------------+
```

mirror positions

Suppose that we make the following call:

```java
makePalindrome(s);
```

Notice that the first and last value are considered to be in mirror positions and they don't match (8 versus 7), so that pair will be removed. The second and second-to-last values are in mirror positions and they match (both 12), so they will not be removed. The third and third-to-last are in mirror positions and they don't match (5 versus 6), so that pair will be removed. The innermost pair matches (both 3), so it won't be removed. Thus, the stack ends up storing the two pairs that match:

```
   bottom [12, 3, 3, 12] top
```

If the stack has an odd length, then the middle value should be retained because it can be part of a palindrome. For example, if the stack stored these values initially:

```
   bottom [1, 2, 3, 4, 3, 7, 1] top
^ ^ ^ ^ ^ ^
| | +-----+ | |
| +----------+ | |
+-------------------+
```

mirror positions

Then after the call, the stack should store the following values:

```
   bottom [1, 3, 4, 3, 1] top
```

Notice that the middle value of 4 has been retained along with the two mirror pairs that match.

You are to use one queue as auxiliary storage to solve this problem. You may not use any other auxiliary data structures to solve this problem, although you can have as many simple variables as you like. You also may not solve the problem recursively. Your solution must run in \(O(n)\) time where \(n\) is the size of the stack. Use the Stack and Queue structures described in the cheat sheet and obey the restrictions described there (recall that you can't use the peek method or a foreach loop or iterator).