Building Java Programs

Chapter 13
binary search and complexity

reading: 13.1-13.2
Let’s write a method to calculate the sum from 1 to some n

```java
public static int sum1(int n) {
    int sum = 0;
    for (int i = 1; i <= n; i++) {
        sum += i;
    }
    return sum;
}
```

Gauss also has a way of solving this

```java
public static int sum2(int n) {
    return n * (n + 1) / 2;
}
```

Which one is more efficient?
Runtime Efficiency (13.2)

- **efficiency**: measure of computing resources used by code.
  - can be relative to speed (time), memory (space), etc.
  - most commonly refers to run time
- We want to be able to compare different algorithms to see which is more efficient
Efficiency Try 1

- Let’s time the methods!

\[
\begin{align*}
n &= 1 & \text{sum1 took } & 0ms, \text{ sum2 took } 0ms \\
n &= 5 & \text{sum1 took } & 0ms, \text{ sum2 took } 0ms \\
n &= 10 & \text{sum1 took } & 0ms, \text{ sum2 took } 0ms \\
n &= 100 & \text{sum1 took } & 0ms, \text{ sum2 took } 0ms \\
n &= 1,000 & \text{sum1 took } & 0ms, \text{ sum2 took } 0ms \\
n &= 10,000,000 & \text{sum1 took } & 18ms, \text{ sum2 took } 0ms \\
n &= 100,000,000 & \text{sum1 took } & 123ms, \text{ sum2 took } 0ms \\
n &= 2,147,483,647 & \text{sum1 took } & 1800ms, \text{ sum2 took } 0ms
\end{align*}
\]

- Downsides
  - Different computers give different run times
  - The same computer gives different results!!! D:<
Efficiency – Try 2

- Count number of “simple steps” our algorithm takes to run
- Assume the following:
  - Any single Java statement takes same amount of time to run.
    - `int x = 5;`
    - `boolean b = (5 + 1 * 2) < 15 + 3;`
    - `return x * 3;`
  - A loop's runtime, if the loop repeats N times, is N times the runtime of the statements in its body.
  - A method call's runtime is measured by the total runtime of the statements inside the method's body.
public static void method1(int N) {
  statement1;
  statement2;
  statement3;

  for (int i = 1; i <= N; i++) {
    statement4;
  }

  for (int i = 1; i <= N; i++) {
    statement5;
    statement6;
    statement7;
  }
}
public static void method2(int N) {
    for (int i = 1; i <= N; i++) {
        for (int j = 1; j <= N; j++) {
            statement1;
        }
    }
}

for (int i = 1; i <= N; i++) {
    statement2;
    statement3;
    statement4;
    statement5;
}

• How many statements will execute if N = 10? If N = 1000?
Sum this up for me

• Let’s write a method to calculate the sum from 1 to some n

```java
public static int sum1(int n) {
    int sum = 0;
    for (int i = 1; i <= n; i++) {
        sum += i;
    }
    return sum;
}
```

• Gauss also has a way of solving this

```java
public static int sum2(int n) {
    return n * (n + 1) / 2;
}
```

• Which one is more efficient?
Visualizing Difference

Comparing $\text{sum1}$ and $\text{sum2}$
Algorithm growth rates (13.2)

- We measure runtime in proportion to the input data size, N.
  - **growth rate**: Change in runtime as N changes.

- Say an algorithm runs $0.4N^3 + 25N^2 + 8N + 17$ statements.
  - Consider the runtime when N is extremely large.
  - We ignore constants like 25 because they are tiny next to N.
  - The highest-order term ($N^3$) dominates the overall runtime.

- We say that this algorithm runs "on the order of" $N^3$.
- or $O(N^3)$ for short ("Big-Oh of N cubed")
• Consider this method:

```java
public void method(int n) {
    int value = 0;
    for (int i = 0; i < 7; i++) {
        for (int j = 0; j < n; j++) {
            value += j;
        }
    }
    return value + n / 2;
}
```

• What is the Big-O efficiency for this method?
  • O(1)
  • O(n)
  • O(7n)
  • O(7n + 4);
  • O(n^2)
  • O(n^3)
**Complexity classes**

- **complexity class**: A category of algorithm efficiency based on the algorithm's relationship to the input size N.

<table>
<thead>
<tr>
<th>Class</th>
<th>Big-Oh</th>
<th>If you double N, ...</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>$O(1)$</td>
<td>unchanged</td>
<td>10ms</td>
</tr>
<tr>
<td>logarithmic</td>
<td>$O(\log_2 N)$</td>
<td>increases slightly</td>
<td>175ms</td>
</tr>
<tr>
<td>linear</td>
<td>$O(N)$</td>
<td>doubles</td>
<td>3.2 sec</td>
</tr>
<tr>
<td>log-linear</td>
<td>$O(N \log_2 N)$</td>
<td>slightly more than doubles</td>
<td>6 sec</td>
</tr>
<tr>
<td>quadratic</td>
<td>$O(N^2)$</td>
<td>quadruples</td>
<td>1 min 42 sec</td>
</tr>
<tr>
<td>cubic</td>
<td>$O(N^3)$</td>
<td>multiplies by 8</td>
<td>55 min</td>
</tr>
<tr>
<td>exponential</td>
<td>$O(2^N)$</td>
<td>multiplies drastically</td>
<td>$5 \times 10^{61}$ years</td>
</tr>
</tbody>
</table>
Complexity classes

Range algorithm

What complexity class is this algorithm? Can it be improved?

// returns the range of values in the given array;
// the difference between elements furthest apart
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int maxDiff = 0; // look at each pair of values
    for (int i = 0; i < numbers.length; i++) {
        for (int j = 0; j < numbers.length; j++) {
            int diff = Math.abs(numbers[j] - numbers[i]);
            if (diff > maxDiff) {
                maxDiff = diff;
            }
        }
    }
    return diff;
}
Range algorithm 2

The last algorithm is $O(N^2)$. A slightly better version:

```java
// returns the range of values in the given array;
// the difference between elements furthest apart
// example: range([17, 29, 11, 4, 20, 8]) is 25
public static int range(int[] numbers) {
    int maxDiff = 0; // look at each pair of values
    for (int i = 0; i < numbers.length; i++) {
        for (int j = i + 1; j < numbers.length; j++) {
            int diff = Math.abs(numbers[j] - numbers[i]);
            if (diff > maxDiff) {
                maxDiff = diff;
            }
        }
    }
    return maxDiff;
}
```
Range algorithm 3

This final version is $O(N)$. It runs MUCH faster:

```java
// returns the range of values in the given array;
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int max = numbers[0];  // find max/min values
    int min = max;
    for (int i = 1; i < numbers.length; i++) {
        if (numbers[i] < min) {
            min = numbers[i];
        }
        if (numbers[i] > max) {
            max = numbers[i];
        }
    }
    return max - min;
}
```
Runtime of first 2 versions

- **Version 1:**

<table>
<thead>
<tr>
<th>N</th>
<th>Runtime (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>15</td>
</tr>
<tr>
<td>2000</td>
<td>47</td>
</tr>
<tr>
<td>4000</td>
<td>203</td>
</tr>
<tr>
<td>8000</td>
<td>781</td>
</tr>
<tr>
<td>16000</td>
<td>3110</td>
</tr>
<tr>
<td>32000</td>
<td>12563</td>
</tr>
<tr>
<td>64000</td>
<td>49937</td>
</tr>
</tbody>
</table>

- **Version 2:**

<table>
<thead>
<tr>
<th>N</th>
<th>Runtime (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>16</td>
</tr>
<tr>
<td>2000</td>
<td>16</td>
</tr>
<tr>
<td>4000</td>
<td>110</td>
</tr>
<tr>
<td>8000</td>
<td>406</td>
</tr>
<tr>
<td>16000</td>
<td>1578</td>
</tr>
<tr>
<td>32000</td>
<td>6265</td>
</tr>
<tr>
<td>64000</td>
<td>25031</td>
</tr>
</tbody>
</table>
Runtime of 3rd version

- Version 3:

<table>
<thead>
<tr>
<th>N</th>
<th>Runtime (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>2000</td>
<td>0</td>
</tr>
<tr>
<td>4000</td>
<td>0</td>
</tr>
<tr>
<td>8000</td>
<td>0</td>
</tr>
<tr>
<td>16000</td>
<td>0</td>
</tr>
<tr>
<td>32000</td>
<td>0</td>
</tr>
<tr>
<td>64000</td>
<td>0</td>
</tr>
<tr>
<td>128000</td>
<td>0</td>
</tr>
<tr>
<td>256000</td>
<td>0</td>
</tr>
<tr>
<td>512000</td>
<td>0</td>
</tr>
<tr>
<td>1e6</td>
<td>0</td>
</tr>
<tr>
<td>2e6</td>
<td>16</td>
</tr>
<tr>
<td>4e6</td>
<td>31</td>
</tr>
<tr>
<td>8e6</td>
<td>47</td>
</tr>
<tr>
<td>1.67e7</td>
<td>94</td>
</tr>
<tr>
<td>3.3e7</td>
<td>188</td>
</tr>
<tr>
<td>6.5e7</td>
<td>453</td>
</tr>
<tr>
<td>1.3e8</td>
<td>797</td>
</tr>
<tr>
<td>2.6e8</td>
<td>1578</td>
</tr>
</tbody>
</table>

Graph showing the runtime (in milliseconds) for different input sizes (N).
• **sequential search**: Locates a target value in an array / list by examining each element from start to finish. Used in `indexOf`.

  • How many elements will it need to examine?
  • Example: Searching the array below for the value 42:

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>-4</td>
<td>2</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>22</td>
<td>25</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>50</td>
<td>56</td>
<td>68</td>
<td>85</td>
<td>92</td>
<td>103</td>
</tr>
</tbody>
</table>

```
Sequential search

• What is its complexity class?

```java
public int indexOf(int value) {
    for (int i = 0; i < size; i++) {
        if (elementData[i] == value) {
            return i;
        }
    }
    return -1;  // not found
}
```

On average, "only" N/2 elements are visited
• 1/2 is a constant that can be ignored

| index | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| value | -4 | 2  | 7  | 10 | 15 | 20 | 22 | 25 | 30 | 36 | 42 | 50 | 56 | 68 | 85 | 92 | 103 |
### Binary search (**13.1**)

- **Binary search**: Locates a target value in a *sorted* array or list by successively eliminating half of the array from consideration.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>-4</td>
<td>2</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>22</td>
<td>25</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>50</td>
<td>56</td>
<td>68</td>
<td>85</td>
<td>92</td>
<td>103</td>
</tr>
</tbody>
</table>

Index: 10

Min: 2

Mid: 22

Max: 103
Binary search

- **binary search** successively eliminates half of the elements.
  
  - **Algorithm:** Examine the middle element of the array.
    - If it is too big, eliminate the right half of the array and repeat.
    - If it is too small, eliminate the left half of the array and repeat.
    - Else it is the value we're searching for, so stop.

- Which indexes does the algorithm examine to find value 42?
- What is the runtime complexity class of binary search?

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>-4</td>
<td>2</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>22</td>
<td>25</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>50</td>
<td>56</td>
<td>68</td>
<td>85</td>
<td>92</td>
<td>103</td>
</tr>
</tbody>
</table>

min  mid  max
Binary search runtime

- For an array of size N, it eliminates \( \frac{1}{2} \) until 1 element remains.
  
  \[ N, \frac{N}{2}, \frac{N}{4}, \frac{N}{8}, \ldots, 4, 2, 1 \]

  - How many divisions does it take?

- Think of it from the other direction:
  
  - How many times do I have to multiply by 2 to reach N?
    
    \[ 1, 2, 4, 8, \ldots, \frac{N}{4}, \frac{N}{2}, N \]

  - Call this number of multiplications "x".

  \[ 2^x = N \]

  \[ x = \log_2 N \]

- Binary search is in the **logarithmic** complexity class.
Complexity classes

Collection efficiency

- Efficiency of our Java's `ArrayList` and `LinkedList` methods:

<table>
<thead>
<tr>
<th>Method</th>
<th><code>ArrayList</code></th>
<th><code>LinkedList</code></th>
</tr>
</thead>
<tbody>
<tr>
<td>add</td>
<td>O(1)*</td>
<td>O(1)</td>
</tr>
<tr>
<td>add(<code>index</code>, <code>value</code>)</td>
<td>O(N)</td>
<td>O(N)</td>
</tr>
<tr>
<td>indexOf</td>
<td>O(N)</td>
<td>O(N)</td>
</tr>
<tr>
<td>get</td>
<td>O(1)</td>
<td>O(N)</td>
</tr>
<tr>
<td>remove</td>
<td>O(N)</td>
<td>O(N)</td>
</tr>
<tr>
<td>set</td>
<td>O(1)</td>
<td>O(N)</td>
</tr>
<tr>
<td>size</td>
<td>O(1)</td>
<td>O(1)</td>
</tr>
</tbody>
</table>

* Most of the time!
Recall two weeks ago when we counted the number of unique words in a file. Our first attempt

```java
public static int uniqueWords(Scanner input) {
    List<String> words = new LinkedList<String>();
    while (input.hasNext()) {
        String word = input.next();
        if (!words.contains(word)) {
            words.add(word);
        }
    }
    return words.size();
}
```
Recall two weeks ago when we counted the number of unique words in a file. Our second attempt
We saw briefly that operations on HashSet are $O(1)$

```java
public static int uniqueWords(Scanner input) {
    Set<String> words = new HashSet<String>();
    while (input.hasNext()) {
        String word = input.next();
        words.add(word);
    }
    return words.size();
}
```