Exercise: Dice roll sum

- Write a method `diceSum` similar to `diceRoll`, but it also accepts a desired sum and prints only arrangements that add up to exactly that sum.

```
diceSum(2, 7);  diceSum(3, 7);
```

```
[1, 6]
[2, 5]
[3, 4]
[4, 3]
[5, 2]
[6, 1]
[1, 1, 5]
[1, 2, 4]
[1, 3, 3]
[1, 4, 2]
[1, 5, 1]
[2, 1, 4]
[2, 2, 3]
[2, 3, 2]
[2, 4, 1]
[3, 1, 3]
[3, 2, 2]
[3, 3, 1]
[4, 1, 2]
[4, 2, 1]
[5, 1, 1]
```
Consider all paths?

<table>
<thead>
<tr>
<th>chosen</th>
<th>available</th>
<th>desired sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>3 dice</td>
<td>5</td>
</tr>
</tbody>
</table>

1 | 2 dice  | 2 | 2 dice  | 3 | 2 dice  | 4 | 2 dice  | 5 | 2 dice  | 6 | 2 dice  |
1, 1 | 1 die   | 1, 2 | 1 die   | 1, 3 | 1 die   | 1, 4 | 1 die   | 1, 5 | 1 die   | 1, 6 | 1 die   |
1, 1, 1 | 1, 1, 2 | 1, 1, 3 | 1, 1, 4 | 1, 1, 5 | 1, 1, 6 | 1, 6, 1 | 1, 6, 2 | ...
Optimizations

- We need not visit every branch of the decision tree.
  - Some branches are clearly not going to lead to success.
  - We can preemptively stop, or **prune**, these branches.

- Inefficiencies in our dice sum algorithm:
  - Sometimes the current sum is already too high.
    - (Even rolling 1 for all remaining dice would exceed the sum.)
  - Sometimes the current sum is already too low.
    - (Even rolling 6 for all remaining dice would not reach the sum.)
  - When finished, the code must compute the sum every time.
    - (1+1+1 = ..., 1+1+2 = ..., 1+1+3 = ..., 1+1+4 = ..., ...)
New decision tree

<table>
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<th>desired sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>3 dice</td>
<td>5</td>
</tr>
</tbody>
</table>

...
The "8 Queens" problem

- Consider the problem of trying to place 8 queens on a chess board such that no queen can attack another queen.

- What are the "choices"?

- How do we "make" or "un-make" a choice?

- How do we know when to stop?
Naive algorithm

- for (each square on board):
  - Place a queen there.
  - Try to place the rest of the queens.
  - Un-place the queen.

- How large is the solution space for this algorithm?
  - $64 \times 63 \times 62 \times \ldots$
Better algorithm idea

- Observation: In a working solution, exactly 1 queen must appear in each row and in each column.

- Redefine a "choice" to be valid placement of a queen in a particular column.

- How large is the solution space now?
  - $8 \times 8 \times 8 \times ...$
Recall: Backtracking

A general pseudo-code algorithm for backtracking problems:

Explore(\texttt{choices}): 

- if there are no more \texttt{choices} to make: stop.
- else, for each available choice \texttt{C}:
  - Choose \texttt{C}.
  - Explore the remaining \texttt{choices}.
  - Un-choose \texttt{C}, if necessary. (backtrack!)
Exercise

• Suppose we have a Board class with these methods:

<table>
<thead>
<tr>
<th>Method/Constructor</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>public Board(int size)</td>
<td>construct empty board</td>
</tr>
<tr>
<td>public boolean isSafe(int row, int column)</td>
<td>true if queen can be safely placed here</td>
</tr>
<tr>
<td>public void place(int row, int column)</td>
<td>place queen here</td>
</tr>
<tr>
<td>public void remove(int row, int column)</td>
<td>remove queen from here</td>
</tr>
<tr>
<td>public String toString()</td>
<td>text display of board</td>
</tr>
</tbody>
</table>

• Write a method solveQueens that accepts a Board as a parameter and tries to place 8 queens on it safely.