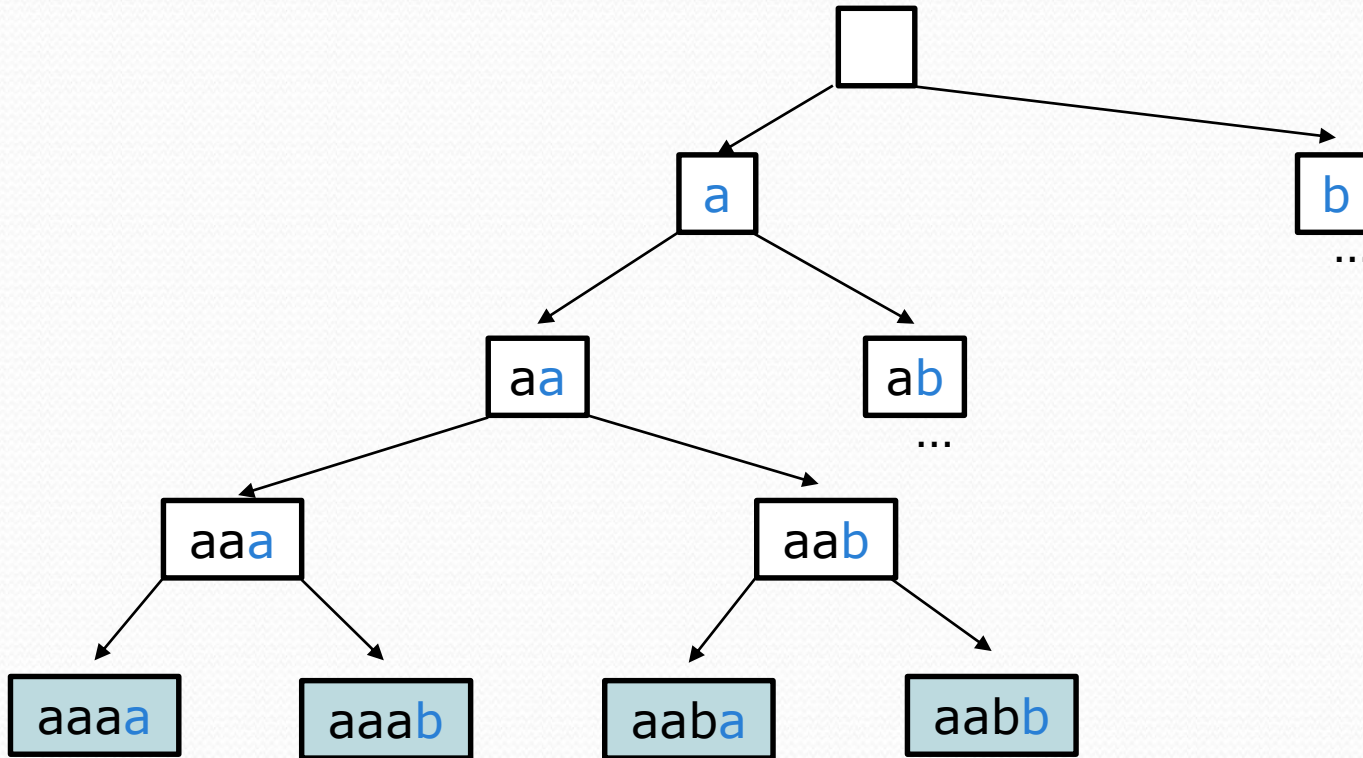


Exercise: fourAB

- Write a method `fourAB` that prints out all strings of length 4 composed only of a's and b's
- Example Output

aaaa	baaa
aaab	baab
aaba	baba
aabb	babb
abaa	bbaa
abab	bbab
abba	bbba
abbb	bbbb

Decision Tree



Exercise: Dice rolls

- Write a method `diceRoll` that accepts an integer parameter representing a number of 6-sided dice to roll, and output all possible arrangements of values that could appear on the dice.

```
diceRoll(2);
```

[1, 1]	[3, 1]	[5, 1]
[1, 2]	[3, 2]	[5, 2]
[1, 3]	[3, 3]	[5, 3]
[1, 4]	[3, 4]	[5, 4]
[1, 5]	[3, 5]	[5, 5]
[1, 6]	[3, 6]	[5, 6]
[2, 1]	[4, 1]	[6, 1]
[2, 2]	[4, 2]	[6, 2]
[2, 3]	[4, 3]	[6, 3]
[2, 4]	[4, 4]	[6, 4]
[2, 5]	[4, 5]	[6, 5]
[2, 6]	[4, 6]	[6, 6]



```
diceRoll(3);
```

[1, 1, 1]
[1, 1, 2]
[1, 1, 3]
[1, 1, 4]
[1, 1, 5]
[1, 1, 6]
[1, 2, 1]
[1, 2, 2]
...
[6, 6, 4]
[6, 6, 5]
[6, 6, 6]

Examining the problem

- We want to generate all possible sequences of values.
for (each possible first die value):
for (each possible second die value):
for (each possible third die value):
...
print!
- This is called a **depth-first search**
- How can we completely explore such a large search space?



A decision tree

chosen	available
-	4 dice

1	3 dice
---	--------

2	3 dice
---	--------

1, 1	2 dice
------	--------

1, 2	2 dice
------	--------

1, 3	2 dice
------	--------

1, 4	2 dice
------	--------

1, 1, 1	1 die
---------	-------

1, 1, 2	1 die
---------	-------

1, 1, 3	1 die
---------	-------

1, 4, 1	1 die
---------	-------

1, 1, 1, 1	
------------	--

1, 1, 1, 2	
------------	--

1, 1, 3, 1	
------------	--

1, 1, 3, 2	
------------	--

Backtracking

- **backtracking**: Finding solution(s) by trying partial solutions and then abandoning them if they are not suitable.
 - a "brute force" algorithmic technique (tries all paths)
 - often implemented recursively

Applications:

- producing all permutations of a set of values
- parsing languages
- games: anagrams, crosswords, word jumbles, 8 queens
- combinatorics and logic programming

Backtracking strategies

- When solving a backtracking problem, ask these questions:
 - What are the "choices" in this problem?
 - What is the "base case"? (How do I know when I'm out of choices?)
 - How do I "make" a choice?
 - Do I need to create additional variables to remember my choices?
 - Do I need to modify the values of existing variables?
 - How do I explore the rest of the choices?
 - Do I need to remove the made choice from the list of choices?
 - Once I'm done exploring, what should I do?
 - How do I "un-make" a choice?

Exercise: Dice roll sum

- Write a method `diceSum` similar to `diceRoll`, but it also accepts a desired sum and prints only arrangements that add up to exactly that sum.

```
diceSum(2, 7);
```

```
[1, 6]  
[2, 5]  
[3, 4]  
[4, 3]  
[5, 2]  
[6, 1]
```

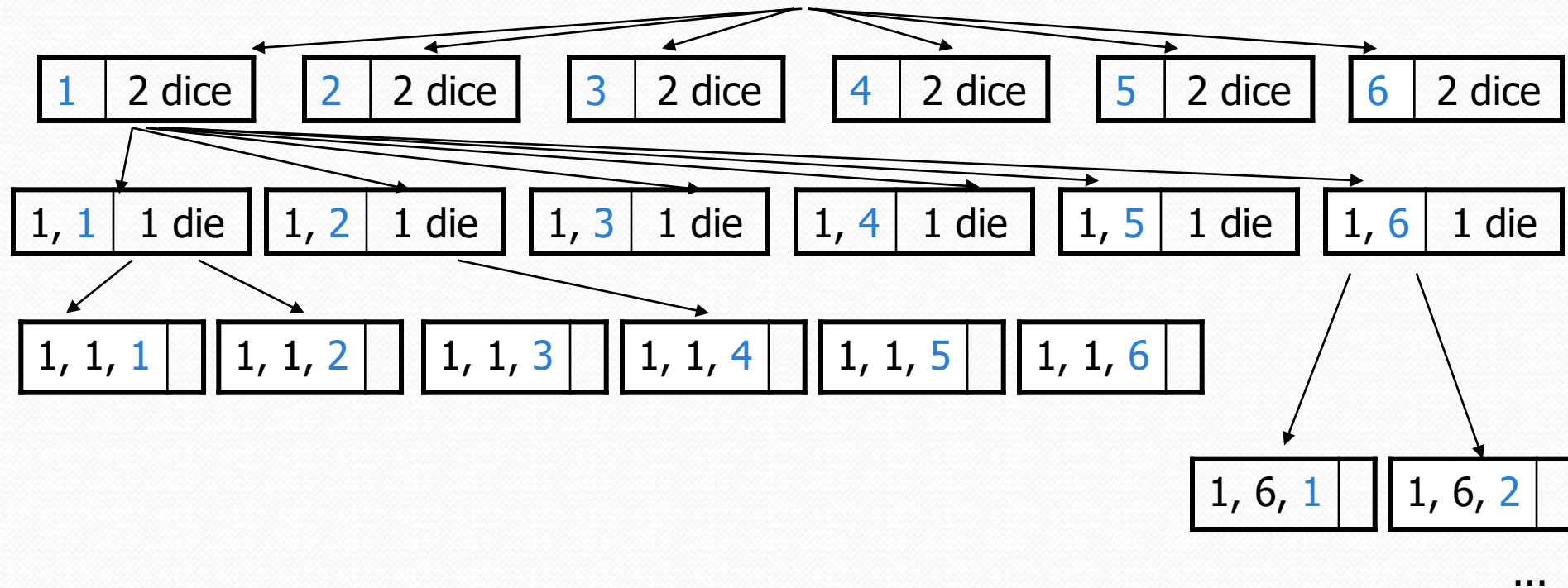


```
diceSum(3, 7);
```

```
[1, 1, 5]  
[1, 2, 4]  
[1, 3, 3]  
[1, 4, 2]  
[1, 5, 1]  
[2, 1, 4]  
[2, 2, 3]  
[2, 3, 2]  
[2, 4, 1]  
[3, 1, 3]  
[3, 2, 2]  
[3, 3, 1]  
[4, 1, 2]  
[4, 2, 1]  
[5, 1, 1]
```


Consider all paths?

chosen	available	desired sum
-	3 dice	5



Optimizations

- We need not visit every branch of the decision tree.
 - Some branches are clearly not going to lead to success.
 - We can preemptively stop, or **prune**, these branches.
- Inefficiencies in our dice sum algorithm:
 - Sometimes the current sum is already too high.
 - (Even rolling 1 for all remaining dice would exceed the sum.)
 - Sometimes the current sum is already too low.
 - (Even rolling 6 for all remaining dice would not reach the sum.)
 - When finished, the code must compute the sum every time.
 - $(1+1+1 = \dots, 1+1+2 = \dots, 1+1+3 = \dots, 1+1+4 = \dots, \dots)$

New decision tree

chosen	available	desired sum
-	3 dice	5

