Building Java Programs

Binary Trees

reading: 17.1 – 17.3
Road Map

**CS Concepts**
- Client/Implementer
- Efficiency
- Recursion
- Regular Expressions
- Grammars
- Sorting
- Backtracking
- Hashing
- Huffman Compression

**Java Language**
- Exceptions
- Interfaces
- References
- Generics
- Comparable
- Inheritance/Polymorphism
- Abstract Classes

**Data Structures**
- Lists
- Stacks
- Queues
- Sets
- Maps
- Priority Queues

**Java Collections**
- Arrays
- ArrayList
- LinkedList
- Stack
- TreeSet / TreeMap
- HashSet / HashMap
- PriorityQueue
Trees in computer science

- TreeMap and TreeSet implementations
- folders/files on a computer
- family genealogy; organizational charts
- AI: decision trees
- compilers: parse tree
  - \( a = (b + c) \times d; \)
- cell phone T9
Trees

- **tree**: Nodes linked together in some hierarchical fashion
- **binary tree**: One where each node has at most two children.

*Recursive definition*: A tree is either:
- empty (*null*), or
- a **root** node that contains:
  - data,
  - a **left** subtree, and
  - a **right** subtree.
  - (The left and/or right subtree could be empty.)
Recursive data structure

- **Recursive definition**: A tree is either:
  - empty (*null*), or
  - a **root** node that contains:
    - *data*,
    - a **left** tree, and
    - a **right** tree
Terminology

- **node**: an object containing a data value and left/right children
  - **root**: topmost node of a tree
  - **leaf**: a node that has no children
  - **branch**: any internal node; neither the root nor a leaf
  - **parent**: a node that refers to this one
  - **child**: a node that this node refers to
  - **sibling**: a node with a common parent
- **subtree**: the smaller tree of nodes on the left or right of the current node
- **height**: length of the longest path from the root to any node
- **level** or **depth**: length of the path from a root to a given node
A tree node for integers

- A basic **tree node object** stores data, refers to left/right
- Multiple nodes can be linked together into a larger tree

<table>
<thead>
<tr>
<th>left</th>
<th>data</th>
<th>right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>left</th>
<th>data</th>
<th>right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>59</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>left</th>
<th>data</th>
<th>right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>left</th>
<th>data</th>
<th>right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>86</td>
<td></td>
</tr>
</tbody>
</table>
// An IntTreeNode object is one node in a binary tree of ints.
public class IntTreeNode {
    public int data; // data stored at this node
    public IntTreeNode left; // reference to left subtree
    public IntTreeNode right; // reference to right subtree

    // Constructs a leaf node with the given data.
    public IntTreeNode(int data) {
        this(data, null, null);
    }

    // Constructs a branch node with the given data and links.
    public IntTreeNode(int data, IntTreeNode left, IntTreeNode right) {
        this.data = data;
        this.left = left;
        this.right = right;
    }
}
IntTree class

// An IntTree object represents an entire binary tree of ints.
public class IntTree {
    private IntTreeNode overallRoot;  // null for an empty tree

    methods
}

- Client code talks to the IntTree, not to the node objects inside it.

- Methods of the IntTree create and manipulate the nodes, their data and links between them.
Print IntTree

- We want to write a method that prints out the contents of an IntTree.
- Here is the output we want

17 41 29 9 81 40

```java
private void print(IntTreeNode root) {
    if (root != null) {
        System.out.print(root.data + " ");
        print(root.left);
        print(root.right);
    }
}
```
Traversals

- Orderings for traversals
  - **pre-order:** process root node, then its left/right subtrees
  - **in-order:** process left subtree, then root node, then right
  - **post-order:** process left/right subtrees, then root node

```java
private void print(IntTreeNode root) {
    if (root != null) {
        System.out.print(root.data + " ");
        print(root.left);
        print(root.right);
    }
}
```

- **pre-order:** 17 41 29 9 81 40
Traversals

- Orderings for traversals
  - **pre-order:** process root node, then its left/right subtrees
  - **in-order:** process left subtree, then root node, then right
  - **post-order:** process left/right subtrees, then root node

private void print(IntTreeNode root) {
    if (root != null) {
        print(root.left);
        System.out.print(root.data + " ");
        print(root.right);
    }
}

- **in-order:** 29 41 17 81 9 40
Traversals

- **Orderings for traversals**
  - **pre-order:** process root node, then its left/right subtrees
  - **in-order:** process left subtree, then root node, then right
  - **post-order:** process left/right subtrees, then root node

```java
private void print(IntTreeNode root) {
    if (root != null) {
        print(root.left);
        print(root.right);
        System.out.print(root.data + " ");
    }
}
```

- **post-order:** 29 41 81 40 9 17
Exercise

- Give pre-, in-, and post-order traversals for the following tree:

- pre: 42 15 27 48 9 86 12 5 3 39
- in: 15 48 27 42 86 5 12 9 3 39
- post: 48 27 15 5 12 86 39 3 42
Traversals trick

- To quickly generate a traversal:
  - Trace a path around the tree.
  - As you pass a node on the proper side, process it.
    - pre-order: left side
    - in-order: bottom
    - post-order: right side

- pre-order: 17 41 29 6 9 81 40
- in-order: 29 41 6 17 81 9 40
- post-order: 29 6 41 81 40 9 17
Exercise

• Add a method `contains` to the `IntTree` class that searches the tree for a given integer, returning `true` if it is found.

  - If an `IntTree` variable `tree` referred to the tree below, the following calls would have these results:

    • `tree.contains(87) → true`
    • `tree.contains(60) → true`
    • `tree.contains(63) → false`
    • `tree.contains(42) → false`
// Returns whether this tree contains the given integer.
public boolean contains(int value) {
    return contains(overallRoot, value);
}

private boolean contains(IntTreeNode node, int value) {
    if (node == null) {
        return false;  // base case: not found here
    } else if (node.data == value) {
        return true;  // base case: found here
    } else {
        // recursive case: search left/right subtrees
        return contains(node.left, value) ||
               contains(node.right, value);
    }
}
Template for tree methods

```java
public class IntTree {
    private IntTreeNode overallRoot;
    ...

    public <type name>(parameters) {
        name(overallRoot, parameters);
    }

    private <type name>(IntTreeNode root, parameters) {
        ...
    }
}
```

- Tree methods are often implemented recursively
  - with a public/private pair
  - the private version accepts the root node to process
Exercise

• Add a method named `printSideways` to the `IntTree` class that prints the tree in a sideways indented format, with right nodes above roots above left nodes, with each level 4 spaces more indented than the one above it.

  – Example: Output from the tree below:
Exercise solution

// Prints the tree in a sideways indented format.
public void printSideways() {
    printSideways(overallRoot, "");
}

private void printSideways(IntTreeNode root, String indent) {
    if (root != null) {
        printSideways(root.right, indent + "    ");
        System.out.println(indent + root.data);
        printSideways(root.left, indent + "    ");
    }
}