Building Java Programs

Chapter 13
binary search and complexity

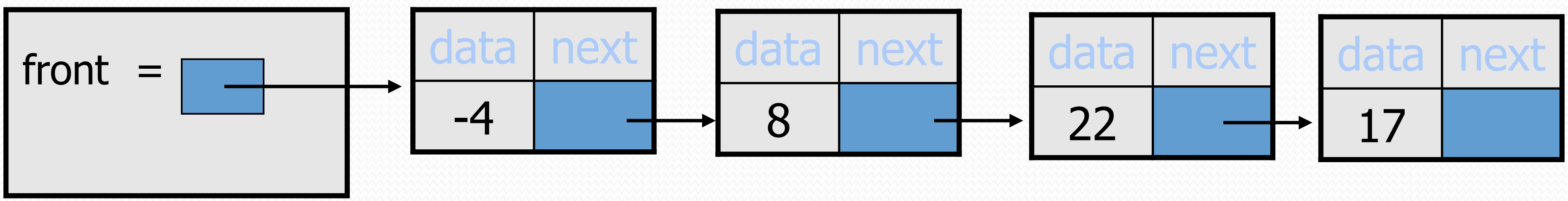
reading: 13.1-13.2

Wednesday Questions

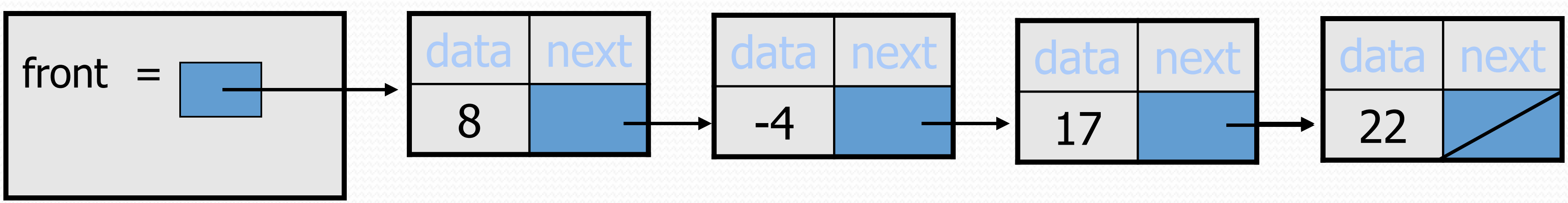
- Are ListNodes used?
  - Yes! In Java's LinkedList
- What does the Stack toString represent?
  - bottom `"[1, 2, 3]"` top
- What program is used for HW specs?
  - LaTeX
- I am a debugging master
  - That's awesome!

switchPairs

- Write a method `switchPairs` that switches each pair of numbers in a list. If there is an odd number of nodes, ignore the last one.
  - Before `switchPairs()` :



  - After `switchPairs()` :



switchPairs

```
public void switchPairs() {
    if (front != null && front.next != null) {
        ListNode current = front.next;
        front.next = current.next;
        current.next = front;
        front = current;
        current = current.next;
        while (current.next != null && current.next.next != null) {
            ListNode temp = current.next.next;
            current.next.next = temp.next;
            temp.next = current.next;
            current.next = temp;
            current = temp.next;
        }
    }
}
```

Sum this up for me

- Let's write a method to calculate the sum from 1 to some n

```
public static int sum1(int n) {
    int sum = 0;
    for (int i = 1; i <= n; i++) {
        sum += i;
    }
    return sum;
}
```

- Gauss also has a way of solving this

```
public static int sum2(int n) {
    return n * (n + 1) / 2;
}
```

- Which one is more efficient?

Runtime Efficiency (13.2)

- **efficiency**: measure of computing resources used by code.
  - can be relative to speed (time), memory (space), etc.
  - most commonly refers to run time
- We want to be able to compare different algorithms to see which is more efficient

Efficiency Try 1

- Let's time the methods!

```
n = 1               sum1 took    0ms,  sum2 took 0ms
n = 5               sum1 took    0ms,  sum2 took 0ms
n = 10              sum1 took    0ms,  sum2 took 0ms
n = 100             sum1 took    0ms,  sum2 took 0ms
n = 1,000           sum1 took    0ms,  sum2 took 0ms
n = 10,000,000      sum1 took   10ms,  sum2 took 0ms
n = 100,000,000     sum1 took  123ms,  sum2 took 0ms
n = 2,147,483,647   sum1 took 1580ms,  sum2 took 0ms
```

- Downsides
  - Different computers give different run times
  - The same computer gives different results!!! D:<

Efficiency – Try 2

- Let's count number of "steps" our algorithm takes to run
- Assume the following:
  - Any single Java statement takes same amount of time to run.
    - `int x = 5;`
    - `boolean b = (5 + 1 * 2) < 15 + 3;`
    - `System.out.println("Hello");`
  - A loop's runtime, if the loop repeats N times, is N times the runtime of the statements in its body.
  - A method call's runtime is measured by the total runtime of the statements inside the method's body.

Efficiency examples

```
statement1;
statement2;
statement3;

for (int i = 1; i <= N; i++) {
    statement4;
}

for (int i = 1; i <= N; i++) {
    statement5;
    statement6;
    statement7;
}
```

statement1–statement3: 3

First loop: N

Second loop: 3N

Total: 4N + 3

Efficiency examples 2

```
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
        statement1;
    }
}

for (int i = 1; i <= N; i++) {
    statement2;
    statement3;
    statement4;
    statement5;
}
```

First loop: N^2

Second loop: $4N$

Total: $N^2 + 4N$

- How many statements will execute if N = 10? If N = 1000?

Sum this up for me

- Let’s write a method to calculate the sum from 1 to some n

```
public static int sum1(int n) {
    int sum = 0;                       } 1
    for (int i = 1; i <= n; i++) {     }
        sum += i;                      } N
    }
    return sum;                        } 1
}
```

N + 2

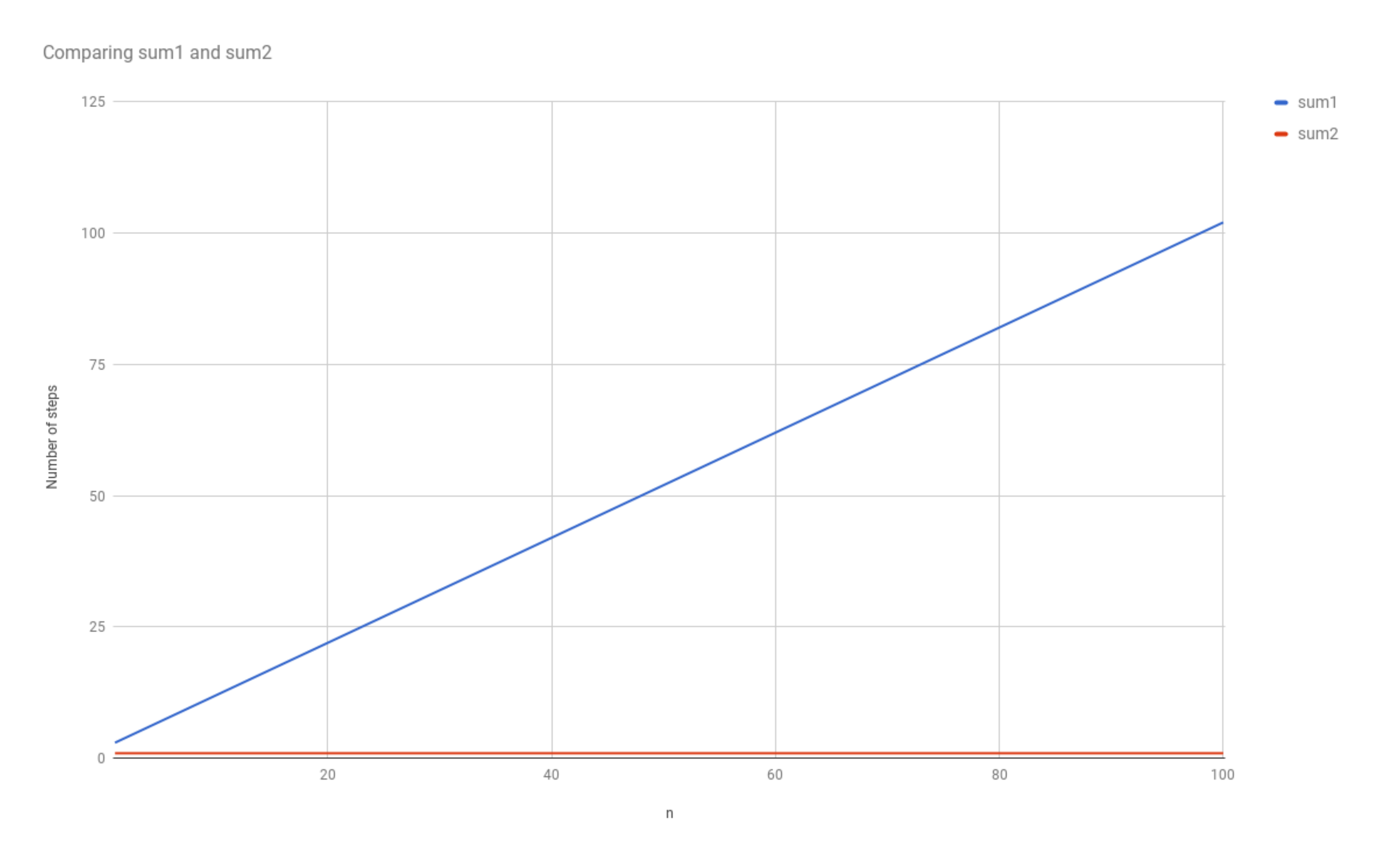
- Gauss also has a way of solving this

```
public static int sum2(int n) {
    return n * (n + 1) / 2;            } 1
}
```

1

- Which one is more efficient?

Visualizing Difference



Algorithm growth rates (13.2)

- We measure runtime in proportion to the input data size, N.
  - **growth rate**: Change in runtime as N changes.

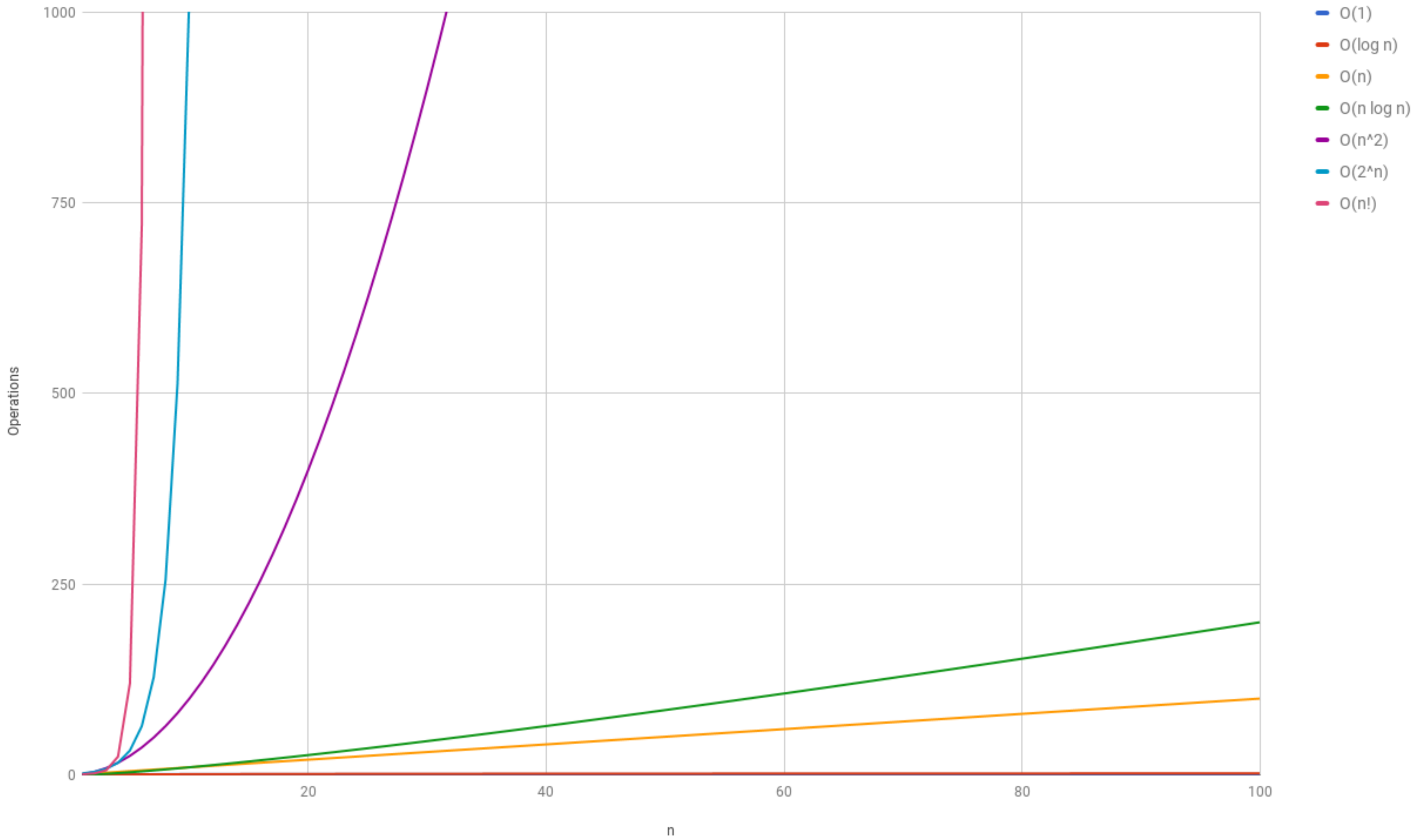
- Say an algorithm runs $\mathbf{0.4N^3 + 25N^2 + 8N + 17}$ statements.
  - Consider the runtime when N is *extremely large* .
  - We ignore constants like 25 because they are tiny next to N.
  - The highest-order term (N^3) dominates the overall runtime.
  - We say that this algorithm runs "on the order of" N^3.
  - or $\mathbf{O(N^3)}$ for short ("**Big-Oh** of N cubed")

Complexity classes

- **complexity class**: A category of algorithm efficiency based on the algorithm's relationship to the input size N.

Class	Big-Oh	If you double N, ...	Example
constant	O(1)	unchanged	10ms
logarithmic	$O(\log_2 N)$	increases slightly	175ms
linear	O(N)	doubles	3.2 sec
log-linear	$O(N \log_2 N)$	slightly more than doubles	6 sec
quadratic	$O(N^2)$	quadruples	1 min 42 sec
cubic	$O(N^3)$	multiplies by 8	55 min
...	...	...	...
exponential	$O(2^N)$	multiplies drastically	$5 * 10^{61}$ years

Complexity classes



http://recursive-design.com/blog/2010/12/07/comp-sci-101-big-o-notation/ - post about a Google interview

Range algorithm

What complexity class is this algorithm? Can it be improved?

```
// returns the range of values in the given array;
// the difference between elements furthest apart
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int maxDiff = 0;     // look at each pair of values
    for (int i = 0; i < numbers.length; i++) {
        for (int j = 0; j < numbers.length; j++) {
            int diff = Math.abs(numbers[j] - numbers[i]);
            if (diff > maxDiff) {
                maxDiff = diff;
            }
        }
    }
    return diff;
}
```

Range algorithm

What complexity class is this algorithm? Can it be improved?

```
// returns the range of values in the given array;
// the difference between elements furthest apart
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int maxDiff = 0;     // look at each pair of values
    for (int i = 0; i < numbers.length; i++) {
        for (int j = 0; j < numbers.length; j++) {
            int diff = Math.abs(numbers[j] - numbers[i]);
            if (diff > maxDiff) {
                maxDiff = diff;
            }
        }
    }
    return diff;
}
```

Range algorithm 2

The last algorithm is **O(N^2)**. A slightly better version:

```
// returns the range of values in the given array;
// the difference between elements furthest apart
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int maxDiff = 0;        // look at each pair of values
    for (int i = 0; i < numbers.length; i++) {
        for (int j = i + 1; j < numbers.length; j++) {
            int diff = Math.abs(numbers[j] - numbers[i]);
            if (diff > maxDiff) {
                maxDiff = diff;
            }
        }
    }
    return diff;
}
```

Range algorithm 3

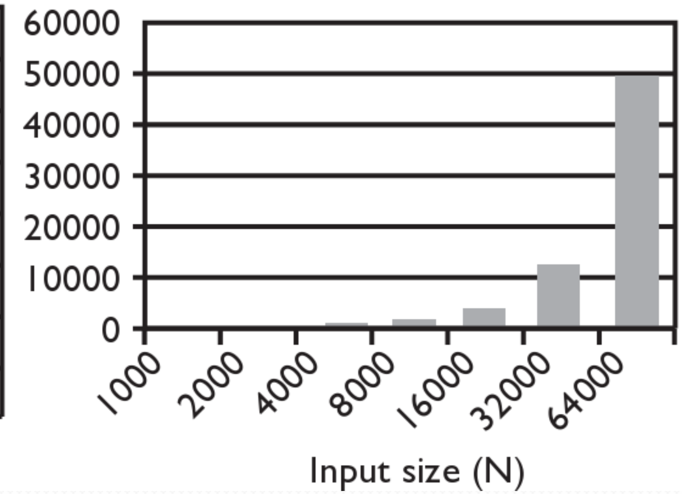
This final version is **O(N)**. It runs MUCH faster:

```
// returns the range of values in the given array;
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int max = numbers[0];       // find max/min values
    int min = max;
    for (int i = 1; i < numbers.length; i++) {
        if (numbers[i] < min) {
            min = numbers[i];
        }
        if (numbers[i] > max) {
            max = numbers[i];
        }
    }
    return max - min;
}
```

Runtime of first 2 versions

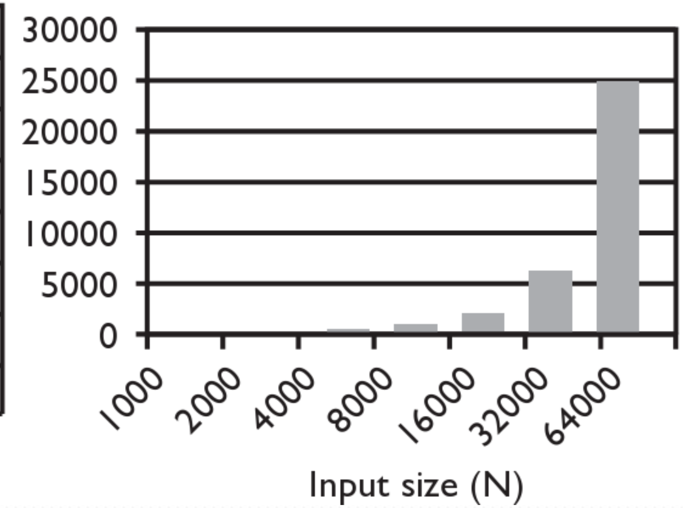
- Version 1:

N	Runtime (ms)
1000	15
2000	47
4000	203
8000	781
16000	3110
32000	12563
64000	49937



- Version 2:

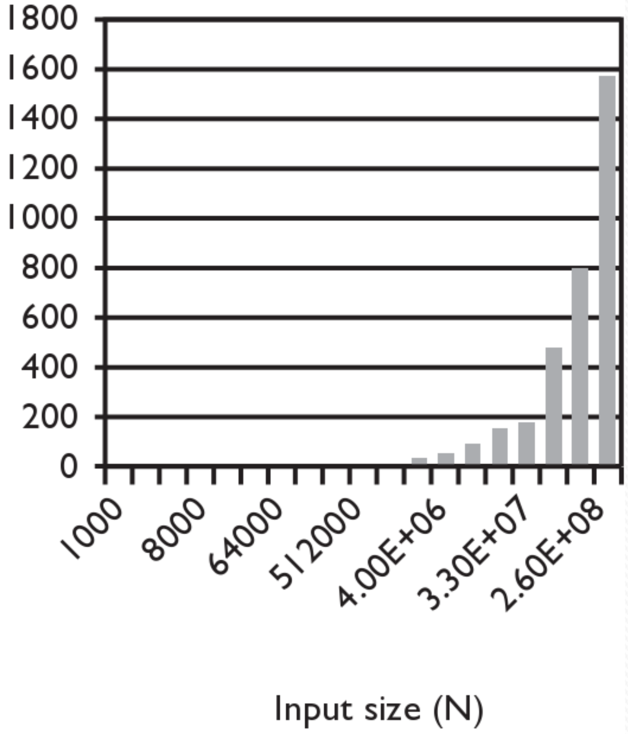
N	Runtime (ms)
1000	16
2000	16
4000	110
8000	406
16000	1578
32000	6265
64000	25031



Runtime of 3rd version

- Version 3:

N	Runtime (ms)
1000	0
2000	0
4000	0
8000	0
16000	0
32000	0
64000	0
128000	0
256000	0
512000	0
1e6	0
2e6	16
4e6	31
8e6	47
1.67e7	94
3.3e7	188
6.5e7	453
1.3e8	797
2.6e8	1578



Searching methods

- Implement the following methods:
  - `indexOf` – returns first index of element, or -1 if not found
  - `contains` - returns true if the list contains the given int value

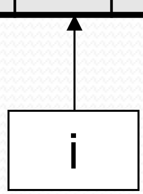
- Why do we need `isEmpty` and `contains` when we already have `indexOf` and `size` ?
  - Adds convenience to the client of our class:

```
// less elegant                         // more elegant
if (myList.size() == 0) {               if (myList.isEmpty()) {
if (myList.indexOf(42) >= 0) {          if (myList.contains(42)) {
```

Sequential search

- **sequential search**: Locates a target value in an array / list by examining each element from start to finish. Used in `indexOf`.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



  - The array is sorted. Could we take advantage of this?

Binary search (13.1)

- **binary search**: Locates a target value in a *sorted* array or list by successively eliminating half of the array from consideration.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

min (index 0) mid (index 8) max (index 16)

Arrays.binarySearch

```
// searches an entire sorted array for a given value
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, value)

// searches given portion of a sorted array for a given value
// examines minIndex (inclusive) through maxIndex (exclusive)
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, minIndex, maxIndex, value)
```

- The `binarySearch` method in the `Arrays` class searches an array very efficiently if the array is sorted.
  - You can search the entire array, or just a range of indexes (useful for "unfilled" arrays such as the one in `ArrayIntList`)

Using `binarySearch`

```
// index     0   1   2   3    4    5    6    7    8    9   10   11   12   13   14   15
int[] a = {-4, 2, 7, 9, 15, 19, 25, 28, 30, 36, 42, 50, 56, 68, 85, 92};

int index  = Arrays.binarySearch(a, 0, 16, 42);   // index1 is 10
int index2 = Arrays.binarySearch(a, 0, 16, 21);   // index2 is -7
```

- `binarySearch` returns the index where the value is found
- if the value is *not* found, `binarySearch` returns:

  `-(insertionPoint + 1)`

  - where `insertionPoint` is the index where the element *would* have been, if it had been in the array in sorted order.
  - To insert the value into the array, negate `insertionPoint` + 1

```
int indexToInsert21 = -(index2 + 1);  // 6
```

Sequential search

- What is its complexity class?

```
public int indexOf(int value) {
    for (int i = 0; i < size; i++) {
        if (elementData[i] == value) {
            return i;
        }
    }
    return -1;   // not found
}
```

N

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

- On average, "only" N/2 elements are visited
  - 1/2 is a constant that can be ignored

Binary search

- **binary search** successively eliminates half of the elements.
  - *Algorithm:* Examine the middle element of the array.
    - If it is too big, eliminate the right half of the array and repeat.
    - If it is too small, eliminate the left half of the array and repeat.
    - Else it is the value we're searching for, so stop.

  - Which indexes does the algorithm examine to find value **42**?
  - What is the runtime complexity class of binary search?

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

min mid max

Binary search runtime

- For an array of size N, it eliminates ½ until 1 element remains.

  N, N/2, N/4, N/8, ..., 4, 2, 1

  - How many divisions does it take?

- Think of it from the other direction:
  - How many times do I have to multiply by 2 to reach N?

    1, 2, 4, 8, ..., N/4, N/2, N
  - Call this number of multiplications "x".

  $2^x = N$

  $\mathbf{x = log_2 N}$

- Binary search is in the **logarithmic** complexity class.

Collection efficiency

- Efficiency of our Java's `ArrayList` and `LinkedList` methods:

Method	**ArrayList**	**LinkedList**
`add`	O(1)*	O(1)
`add(`**index**`, `**value**`)`	O(N)	O(N)
`indexOf`	O(N)	O(N)
`get`	O(1)	O(N)
`remove`	O(N)	O(N)
`set`	O(1)	O(N)
`size`	O(1)	O(1)

* Most of the time!

Max subsequence sum

- Write a method `maxSum` to find the largest sum of any contiguous subsequence in an array of integers.
  - Easy for all positives: include the whole array.
  - What if there are negatives?

index	0	1	2	3	4	5	6	7	8
value	2	1	-4	10	15	-2	22	-8	5

Largest sum: 10 + 15 + -2 + 22 = 45

  - (Let's define the max to be 0 if the array is entirely negative.)

- Ideas for algorithms?

Algorithm 1 pseudocode

```
maxSum(a):
    max = 0.
    for each starting index i:
        for each ending index j:
            sum = add the elements from a[i] to a[j].
            if sum > max,
                max = sum.

    return max.
```

index	0	1	2	3	4	5	6	7	8
value	2	1	-4	10	15	-2	22	-8	5

Algorithm 1 code

- What complexity class is this algorithm?
  - **O(N^3)**. Takes a few seconds to process 2000 elements.

```
public static int maxSum1(int[] a) {
    int max = 0;
    for (int i = 0; i < a.length; i++) {
        for (int j = i; j < a.length; j++) {
            // sum = add the elements from a[i] to a[j].
            int sum = 0;
            for (int k = i; k <= j; k++) {
                sum += a[k];
            }
            if (sum > max) {
                max = sum;
            }
        }
    }
    return max;
}
```

Flaws in algorithm 1

- Observation: We are redundantly re-computing sums.
  - For example, we compute the sum between indexes 2 and 5:
    a[2] + a[3] + a[4] + a[5]
  - Next we compute the sum between indexes 2 and 6:
    a[2] + a[3] + a[4] + a[5] + a[6]
  - We already had computed the sum of 2-5, but we compute it again as part of the 2-6 computation.
  - Let's write an improved version that avoids this flaw.

index	0	1	2	3	4	5	6	7	8
value	2	1	-4	10	15	-2	22	-8	5

Algorithm 2 code

- What complexity class is this algorithm?
  - **$O(N^2)$**. Can process tens of thousands of elements per second.

```
public static int maxSum2(int[] a) {
    int max = 0;
    for (int i = 0; i < a.length; i++) {
        int sum = 0;
        for (int j = i; j < a.length; j++) {
            sum += a[j];
            if (sum > max) {
                max = sum;
            }
        }
    }
    return max;
}
```

index	0	1	2	3	4	5	6	7	8
value	2	1	-4	10	15	-2	22	-8	5

A clever solution

- *Claim 1* : A max range cannot start with a negative-sum range.

i ... j	j+1 ... k
< 0	sum(j+1, k)
sum(i, k) < sum(j+1, k)	

- *Claim 2* : If sum(i, j-1) ≥ 0 and sum(i, j) < 0, any max range that ends at j+1 or higher cannot start at any of i through j.

i ... j-1	j	j+1 ... k
≥ 0	< 0	sum(j+1, k)
< 0		sum(j+1, k)
		sum(?, k) < sum(j+1, k)

  - Together, these observations lead to a very clever algorithm...

Algorithm 3 code

- What complexity class is this algorithm?
  - **O(N)**. Handles many millions of elements per second!

```
public static int maxSum3(int[] a) {
    int max = 0;
    int sum = 0;
    int i = 0;
    for (int j = 0; j < a.length; j++) {
        if (sum < 0) {    // if sum becomes negative, max range
            i = j;        // cannot start with any of i - j-1
            sum = 0;      // (Claim 2)
        }
        sum += a[j];
        if (sum > max) {
            max = sum;
        }
    }
    return max;
}
```