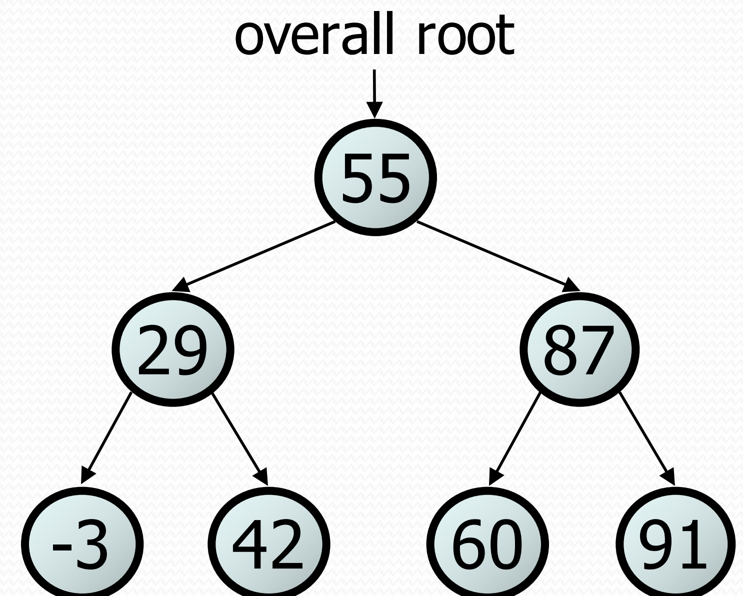
Binary Search Trees

reading: 17.3 – 17.4

Binary search trees

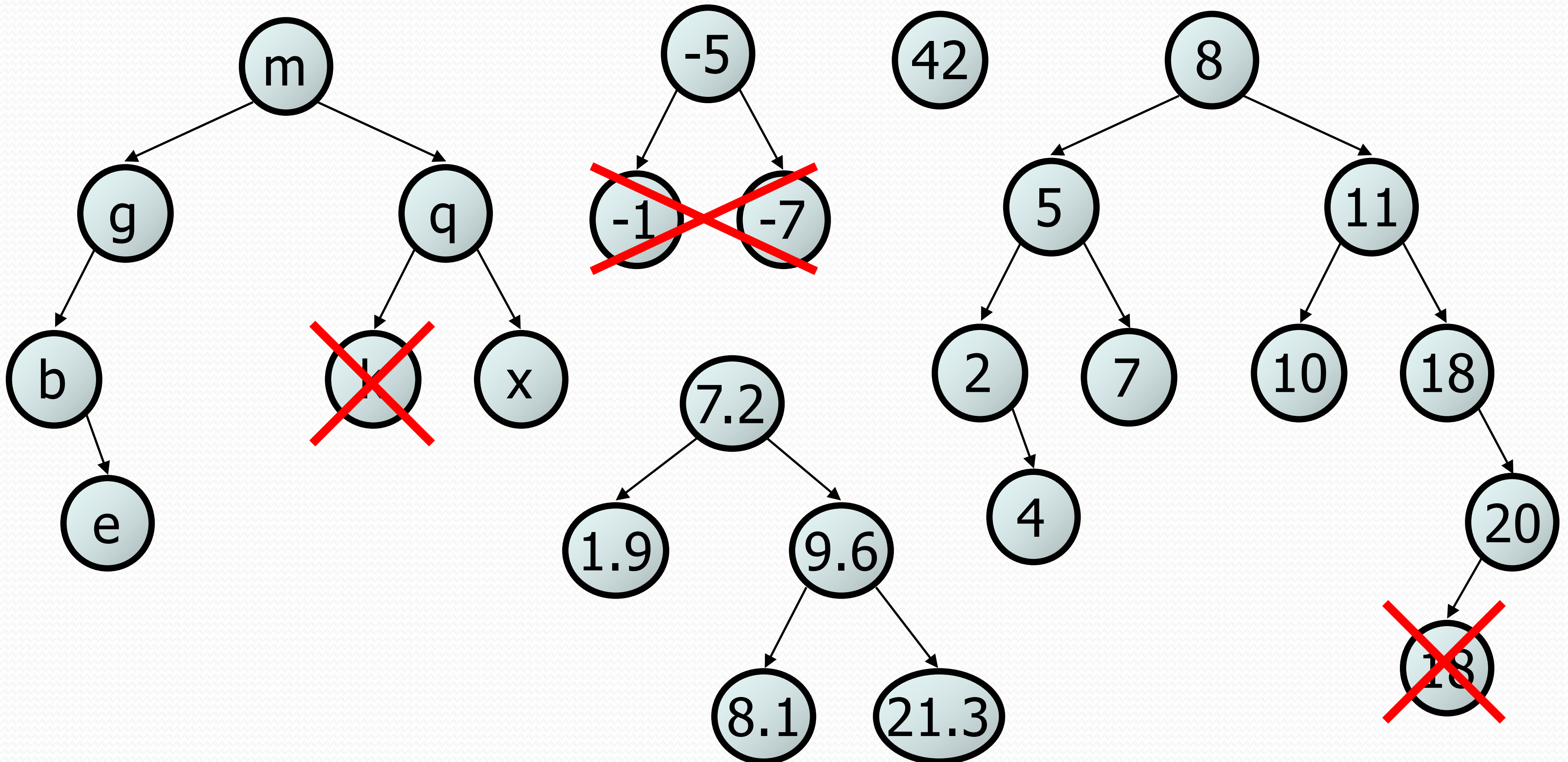
- **binary search tree** ("BST"): a binary tree where each non-empty node R has the following properties:
  - elements of R's left subtree contain data "less than" R's data,
  - elements of R's right subtree contain data "greater than" R's,
  - R's left and right subtrees are also binary search trees.

- BSTs store their elements in sorted order, which is helpful for searching/sorting tasks.



BST examples

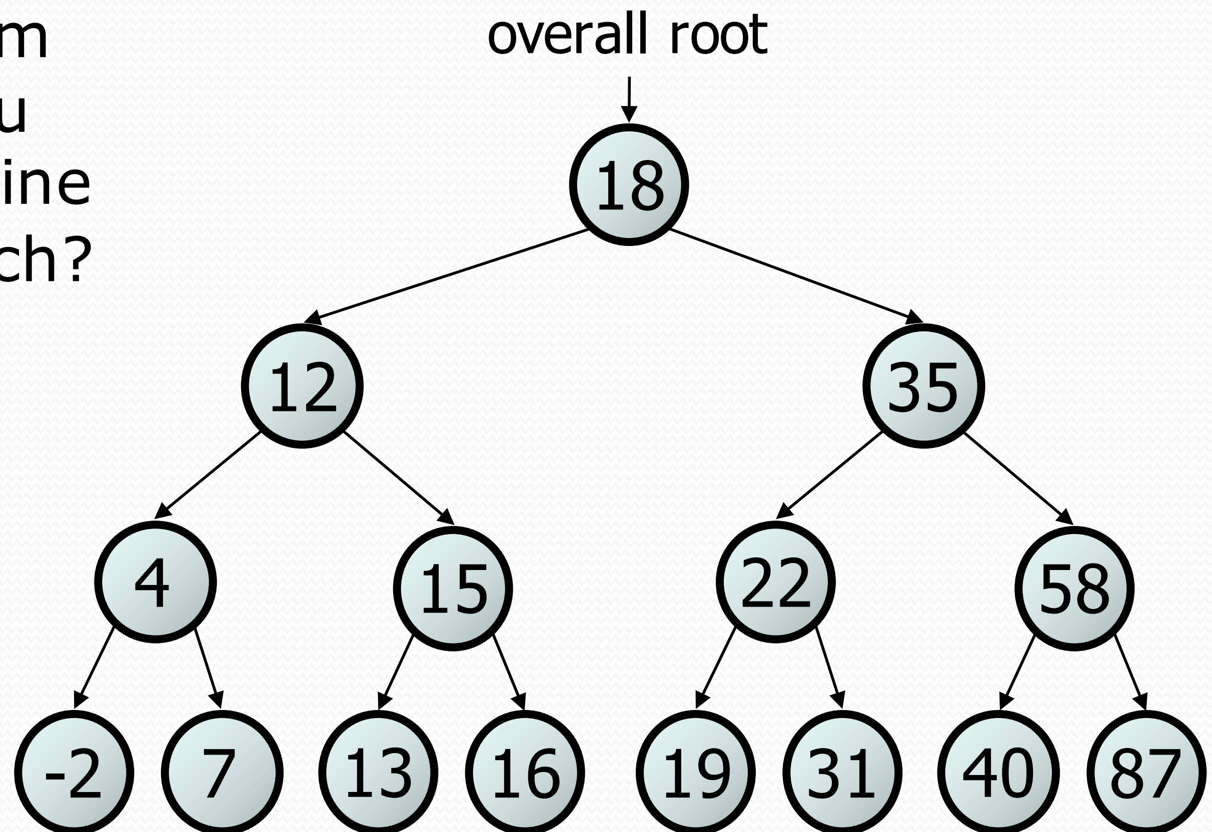
- Which of the trees shown are legal binary search trees?



Searching a BST

- Describe an algorithm for searching a binary search tree.
  - Try searching for the value 31, then 6.

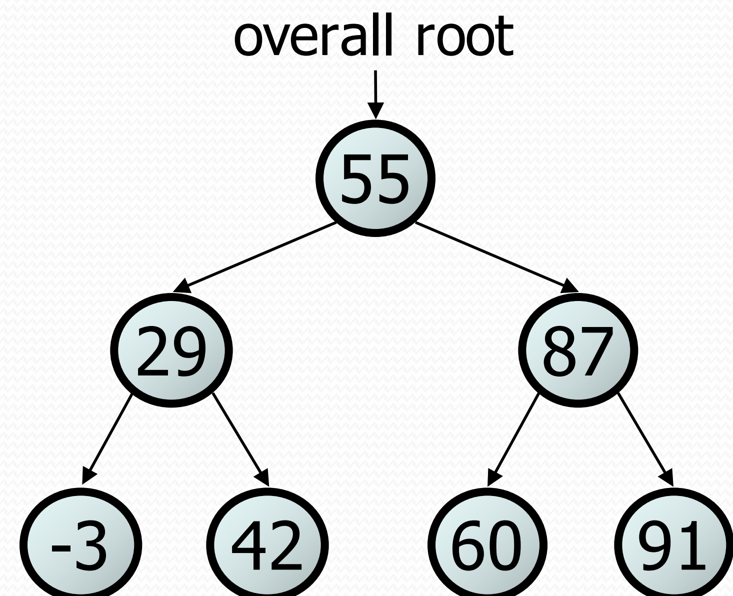
- What is the maximum number of nodes you would need to examine to perform any search?



Exercise

- Convert the `IntTree` class into a `SearchTree` class.
  - The elements of the tree will form a legal binary search tree.

- Write a `contains` method that takes advantage of the BST structure.
  - `tree.contains(29)` → `true`
  - `tree.contains(55)` → `true`
  - `tree.contains(63)` → `false`
  - `tree.contains(35)` → `false`



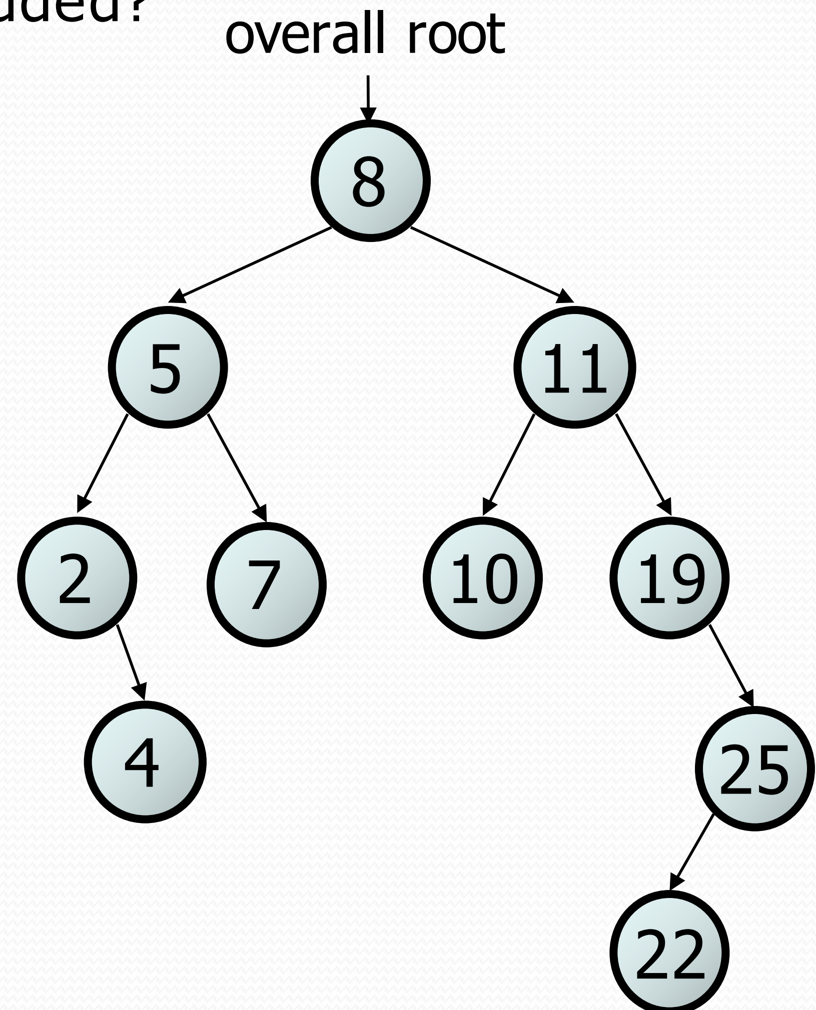
Exercise solution

```
// Returns whether this BST contains the given integer.
public boolean contains(int value) {
    return contains(overallRoot, value);
}

private boolean contains(IntTreeNode node, int value) {
    if (node == null) {
        return false;   // base case: not found here
    } else if (node.data == value) {
        return true;    // base case: found here
    } else if (node.data > value) {
        return contains(node.left, value);
    } else {    // root.data < value
        return contains(node.right, value);
    }
}
```

Adding to a BST

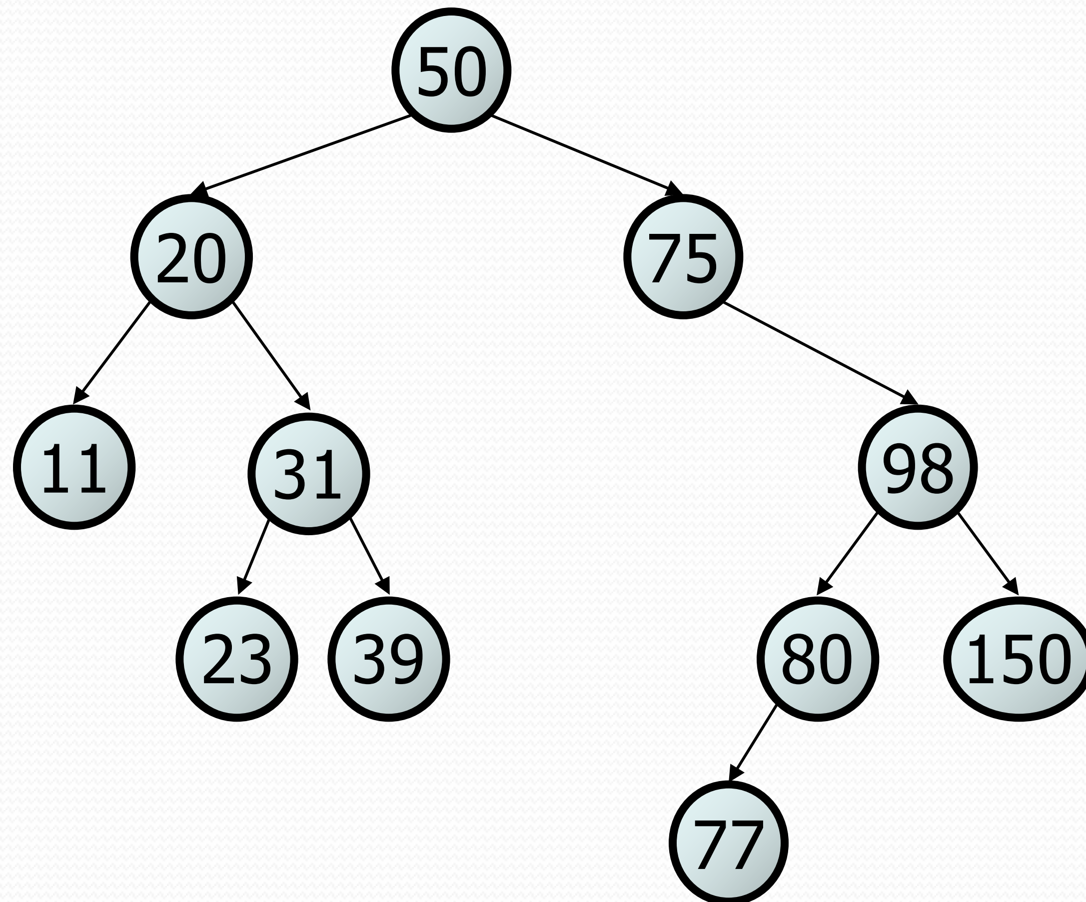
- Suppose we want to add new values to the BST below.
  - Where should the value 14 be added?
  - Where should 3 be added? 7?
  - If the tree is empty, where should a new value be added?
- What is the general algorithm?



Adding exercise

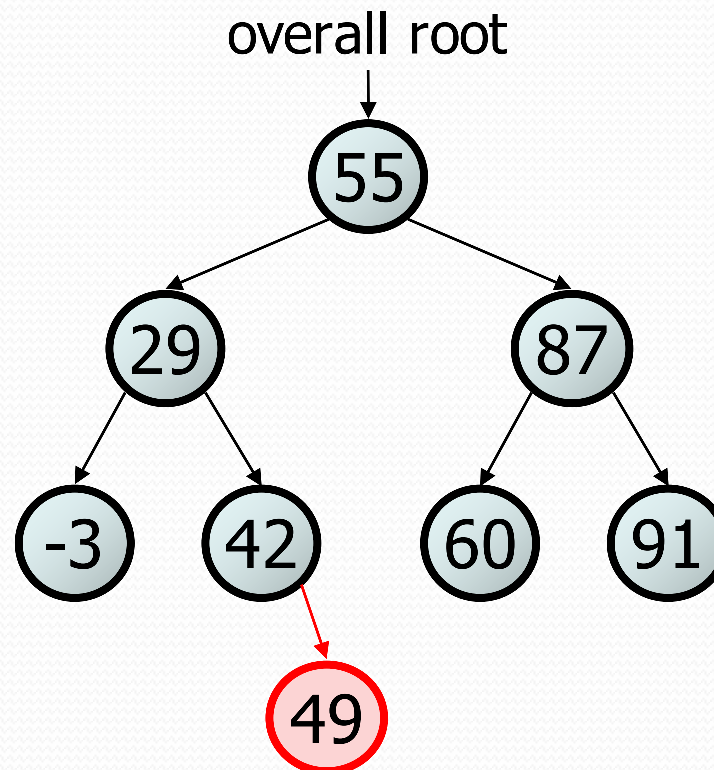
- Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

50
20
75
98
80
31
150
39
23
11
77



Exercise

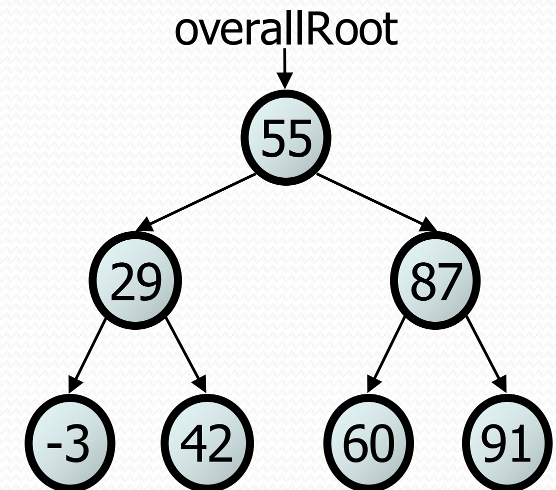
- Add a method `add` to the `SearchTree` class that adds a given integer value to the BST.
  - Add the new value in the proper place to maintain BST ordering.
    - `tree.add(49);`



An incorrect solution

```
// Adds the given value to this BST in sorted order.
public void add(int value) {
    add(overallRoot, value);
}

private void add(IntTreeNode node, int value) {
    if (node == null) {
        node = new IntTreeNode(value);
    } else if (node.data > value) {
        add(node.left, value);
    } else if (node.data < value) {
        add(node.right, value);
    }
    // else node.data == value, so
    // it's a duplicate (don't add)
}
```



- Why doesn't this solution work?

The x = change(x) pattern

read 17.3

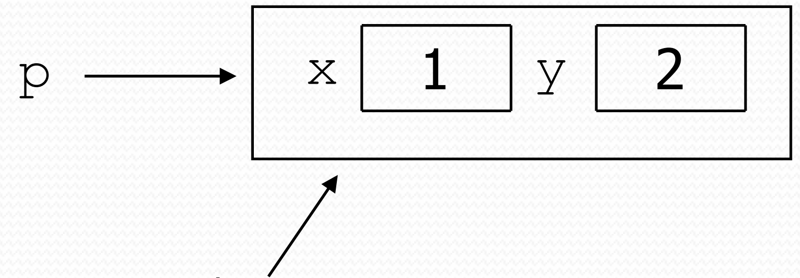
A tangent: Change a point

- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
     Point p = new Point(1, 2);
     change(p);
     System.out.println(p);
}

public static void change(Point thePoint) {
     thePoint.x = 3;
     thePoint.y = 4;
}

// answer: (3, 4)
```



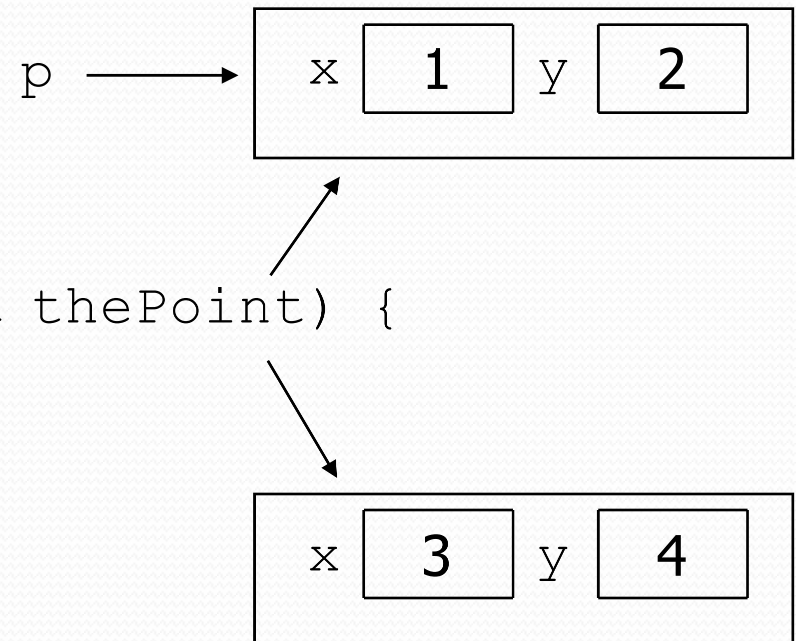
Change point, version 2

- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
    Point p = new Point(1, 2);
    change(p);
    System.out.println(p);
}

public static void change(Point thePoint) {
    thePoint = new Point(3, 4);
}

// answer: (1, 2)
```



Changing references

- If a method *dereferences a variable* (with `.`) and modifies the object it refers to, that change will be seen by the caller.

```
public static void change(Point thePoint) {
    thePoint.x = 3;                    // affects p
    thePoint.setY(4);                  // affects p
```

- If a method *reassigns a variable to refer to a new object,* that change will *not* affect the variable passed in by the caller.

```
public static void change(Point thePoint) {
    thePoint = new Point(3, 4);        // p unchanged
    thePoint = null;                   // p unchanged
```

  - What if we want to make the variable passed in become `null`?

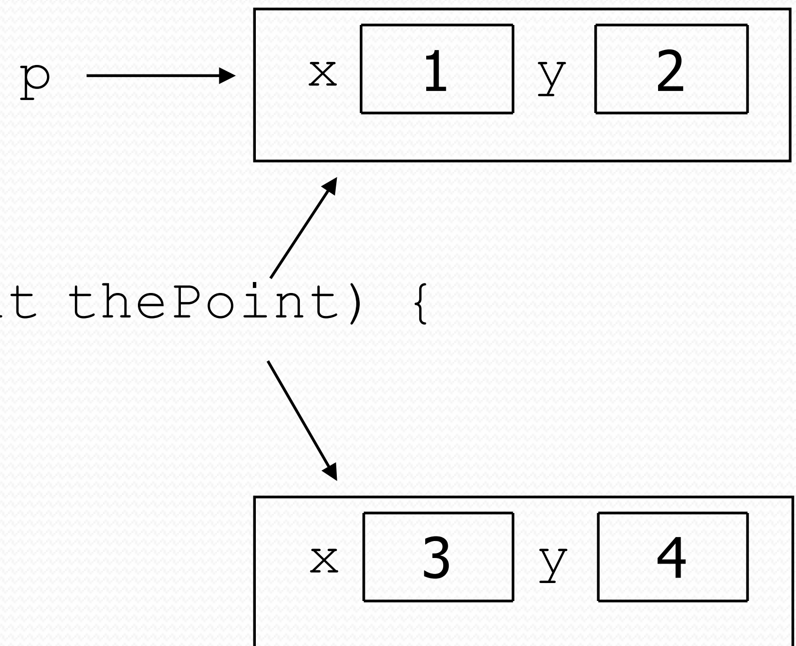
Change point, version 3

- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
    Point p = new Point(1, 2);
    change(p);
    System.out.println(p);
}

public static Point change(Point thePoint) {
    thePoint = new Point(3, 4);
    return thePoint;
}

// answer: (1, 2)
```



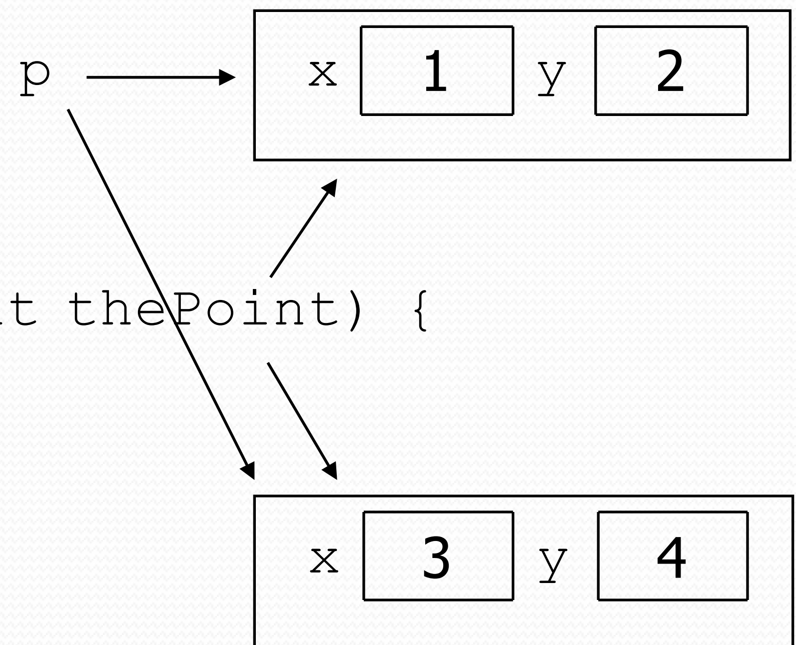
Change point, version 4

- What is the state of the object referred to by `p` after this code?

```
public static void main(String[] args) {
    Point p = new Point(1, 2);
    p = change(p);
    System.out.println(p);
}

public static Point change(Point thePoint) {
    thePoint = new Point(3, 4);
    return thePoint;
}

// answer: (3, 4)
```



x = change(x);

- If you want to write a method that can change the object that a variable refers to, you must do three things:
  1. **pass** in the original state of the object to the method
  2. **return** the new (possibly changed) object from the method
  3. **re-assign** the caller's variable to store the returned result

```
p = change(p);    // in main

public static Point change(Point thePoint) {
    thePoint = new Point(99, -1);
    return thePoint;
```

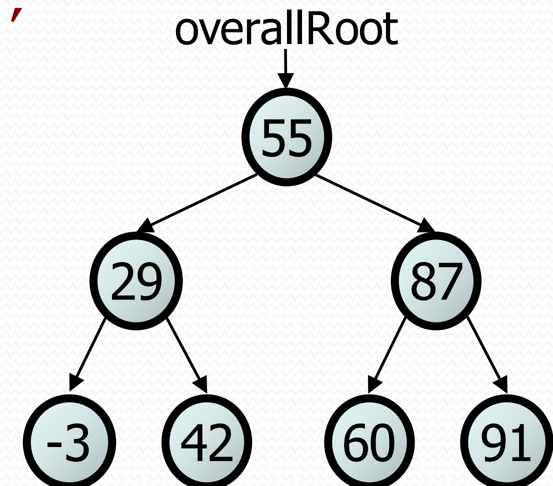
- We call this general algorithmic pattern **x = change(x);**
  - also seen with strings: `s = s.toUpperCase();`

The problem

- Much like with linked lists, if we just modify what a local variable refers to, it won't change the collection.

node → 49

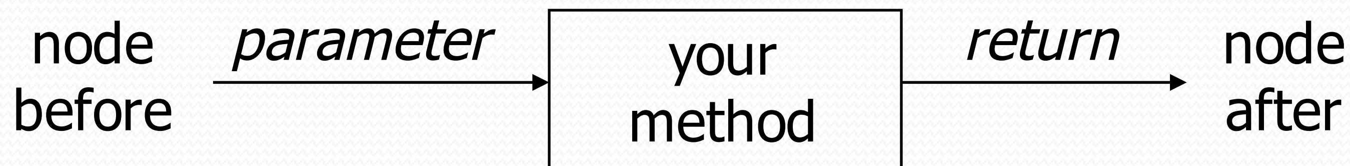
```
private void add(IntTreeNode node, int value) {
    if (node == null) {
        node = new IntTreeNode(value);
    }
```



  - In the linked list case, how did we actually modify the list?
    - by changing the `front`
    - by changing a node's `next` field

Applying x = change(x)

- Methods that modify a tree should have the following pattern:
  - input (parameter): old state of the node
  - output (return): new state of the node



- In order to actually change the tree, you must reassign:

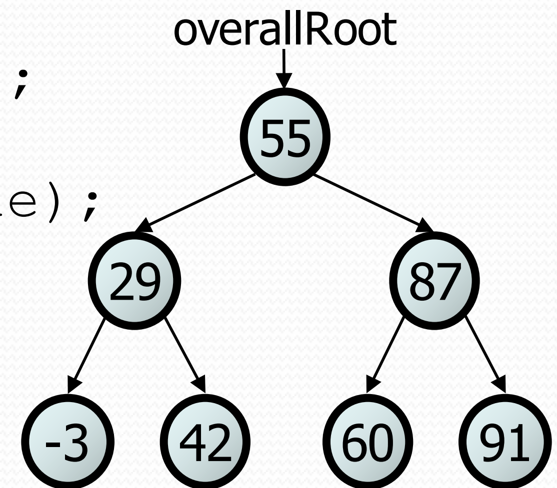
```
node        = change(node, parameters);
node.left   = change(node.left, parameters);
node.right  = change(node.right, parameters);
overallRoot = change(overallRoot, parameters);
```

A correct solution

```
// Adds the given value to this BST in sorted order.
public void add(int value) {
    overallRoot = add(overallRoot, value);
}

private IntTreeNode add(IntTreeNode node, int value) {
    if (node == null) {
        node = new IntTreeNode(value);
    } else if (node.data > value) {
        node.left = add(node.left, value);
    } else if (node.data < value) {
        node.right = add(node.right, value);
    } // else a duplicate; do nothing

    return node;
}
```



- What happens when `node` is a leaf?