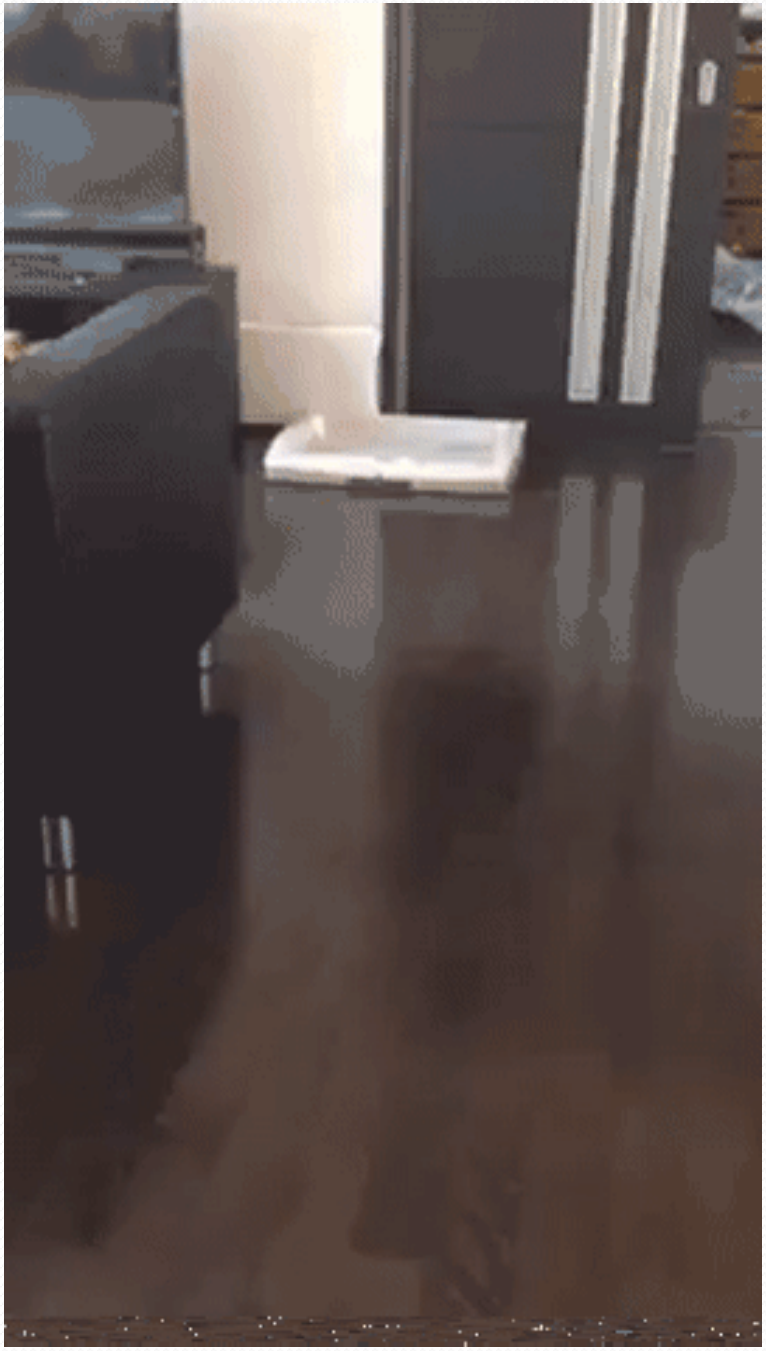
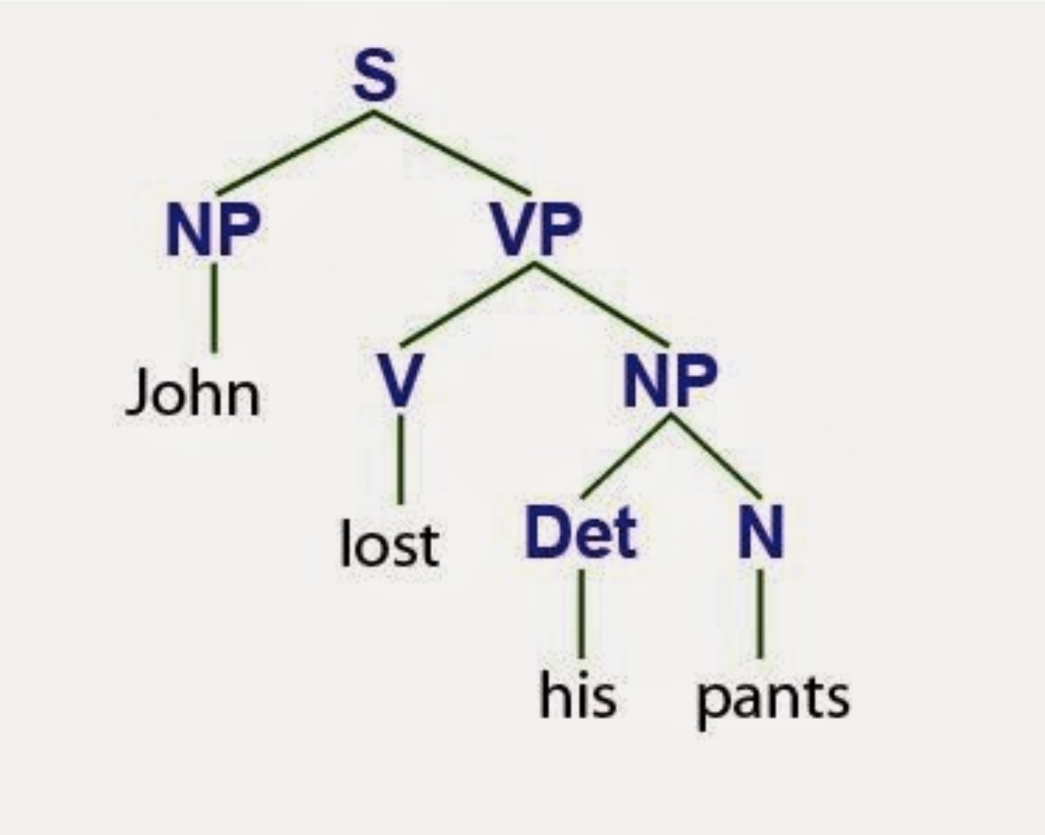
Building Java Programs

Chapter 13
Sorting

reading: 13.3, 13.4



Languages and Grammars


S
NP
VP
John
V
NP
lost
Det
N
his
pants

Languages and grammars

- (formal) **language**: A set of words or symbols.

- **grammar**: A description of a language that describes which sequences of symbols are allowed in that language.
  - describes language *syntax* (rules) but not *semantics* (meaning)
  - can be used to generate strings from a language, or to determine whether a given string belongs to a given language

Backus-Naur (BNF)

- **Backus-Naur Form (BNF)**: A syntax for describing language grammars in terms of transformation *rules*, of the form:

  <**symbol**> ::= <**expression**> | <**expression**> ... | <**expression**>

  - **terminal**: A fundamental symbol of the language.
  - **non-terminal**: A high-level symbol describing language syntax, which can be transformed into other non-terminal or terminal symbol(s) based on the rules of the grammar.

  - developed by two Turing-award-winning computer scientists in 1960 to describe their new ALGOL programming language

An example BNF grammar

```
<s>::=<n> <v>
<n>::=Marty | Victoria | Stuart | Jessica
<v>::=cried | slept | belched
```

- Some sentences that could be generated from this grammar:

```
Marty slept
Jessica belched
Stuart cried
```

BNF grammar version 2

```
<s>::=<np> <v>
<np>::=<pn> | <dp> <n>
<pn>::=Marty | Victoria | Stuart | Jessica
<dp>::=a | the
<n>::=ball | hamster | carrot | computer
<v>::=cried | slept | belched
```

- Some sentences that could be generated from this grammar:

```
the carrot cried
Jessica belched
a computer slept
```

BNF grammar version 3

```
<s>::=<np> <v>
<np>::=<pn> | <dp> <adj> <n>
<pn>::=Marty | Victoria | Stuart | Jessica
<dp>::=a | the
<adj>::=silly | invisible | loud | romantic
<n>::=ball | hamster | carrot | computer
<v>::=cried | slept | belched
```

- Some sentences that could be generated from this grammar:

```
the invisible carrot cried
Jessica belched
a computer slept
a romantic ball belched
```

Grammars and recursion

```
<s>::=<np> <v>
<np>::=<pn> | <dp> <adjp> <n>
<pn>::=Marty | Victoria | Stuart | Jessica
<dp>::=a | the
<adjp>::=<adj> <adjp> | <adj>
<adj>::=silly | invisible | loud | romantic
<n>::=ball | hamster | carrot | computer
<v>::=cried | slept | belched
```

- Grammar rules can be defined *recursively*, so that the expansion of a symbol can contain that same symbol.
  - There must also be expressions that expand the symbol into something non-recursive, so that the recursion eventually ends.

Grammar, final version

```
<s>::=<np> <vp>
<np>::=<dp> <adjp> <n>|<pn>
<dp>::=the|a
<adjp>::=<adj>|<adj> <adjp>
<adj>::=big|fat|green|wonderful|faulty|subliminal
<n>::=dog|cat|man|university|father|mother|child
<pn>::=John|Jane|Sally|Spot|Fred|Elmo
<vp>::=<tv> <np>|<iv>
<tv>::=hit|honored|kissed|helped
<iv>::=died|collapsed|laughed|wept
```

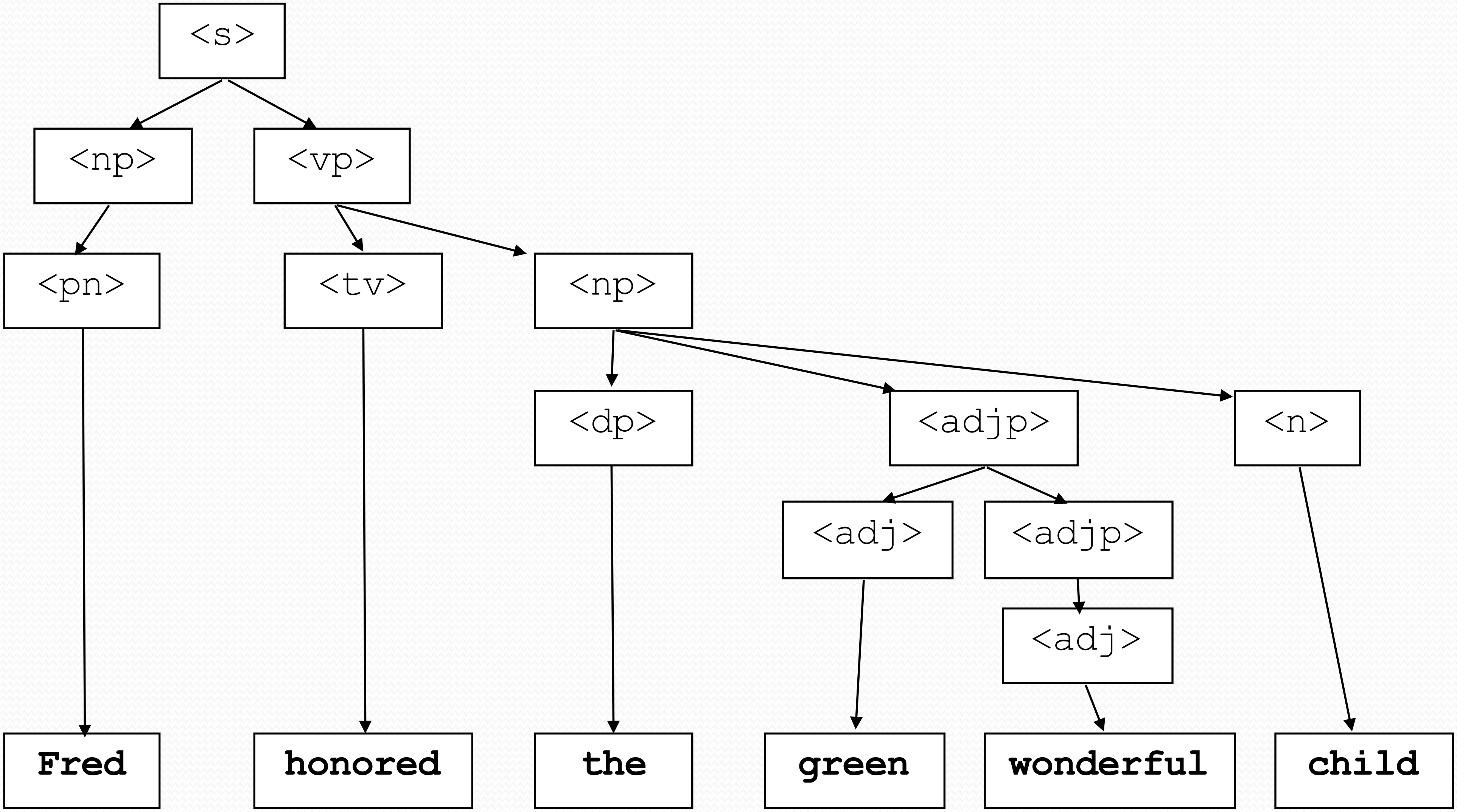
- Could this grammar generate the following sentences?

  `Fred honored the green wonderful child`

  `big Jane wept the fat man fat`

- Generate a random sentence using this grammar.

Sentence generation



<s>

<np> <vp>

<pn> <tv> <np>

<dp> <adjp> <n>

<adj> <adjp>

<adj>

Fred **honored** **the** **green** **wonderful** **child**

`Collections` class

Method name	Description
`binarySearch(`**list**`, `**value**`)`	returns the index of the given value in a sorted list (< 0 if not found)
`copy(`**listTo**`, `**listFrom**`)`	copies **listFrom**'s elements to **listTo**
`emptyList(),emptyMap(),` `emptySet()`	returns a read-only collection of the given type that has no elements
`fill(`**list**`, `**value**`)`	sets every element in the list to have the given value
`max(`**collection**`),min(`**collection**`)`	returns largest/smallest element
`replaceAll(`**list**`, `**old**`, `**new**`)`	replaces an element value with another
`reverse(`**list**`)`	reverses the order of a list's elements
`shuffle(`**list**`)`	arranges elements into a random order
`sort(`**list**`)`	arranges elements into ascending order

Sorting

- **sorting**: Rearranging the values in an array or collection into a specific order (usually into their "natural ordering").
  - one of the fundamental problems in computer science
  - can be solved in many ways:
    - there are many sorting algorithms
    - some are faster/slower than others
    - some use more/less memory than others
    - some work better with specific kinds of data
    - some can utilize multiple computers / processors, ...

  - *comparison-based sorting* : determining order by comparing pairs of elements:
    - <, >, `compareTo`, ...

Sorting methods in Java

- The `Arrays` and `Collections` classes in `java.util` have a static method `sort` that sorts the elements of an array/list

```
String[] words = {"foo", "bar", "baz", "ball"};
Arrays.sort(words);
System.out.println(Arrays.toString(words));
// [ball, bar, baz, foo]

List<String> words2 = new ArrayList<String>();
for (String word : words) {
    words2.add(word);
}
Collections.sort(words2);
System.out.println(words2);
// [ball, bar, baz, foo]
```

Sorting algorithms

- **bogo sort**: shuffle and pray
- **bubble sort**: swap adjacent pairs that are out of order
- **selection sort**: look for the smallest element, move to front
- **insertion sort**: build an increasingly large sorted front portion
- **merge sort**: recursively divide the array in half and sort it
- **heap sort**: place the values into a sorted tree structure
- **quick sort**: recursively partition array based on a middle value

other specialized sorting algorithms:

- **bucket sort**: cluster elements into smaller groups, sort them
- **radix sort**: sort integers by last digit, then 2nd to last, then ...
- ...

Bogo sort

- **bogo sort**: Orders a list of values by repetitively shuffling them and checking if they are sorted.
  - name comes from the word "bogus"

  The algorithm:
  - Scan the list, seeing if it is sorted. If so, stop.
  - Else, shuffle the values in the list and repeat.

- This sorting algorithm (obviously) has terrible performance!
  - What is its runtime?

Bogo sort code

```
// Places the elements of a into sorted order.
public static void bogoSort(int[] a) {
    while (!isSorted(a)) {
        shuffle(a);
    }
}

// Returns true if a's elements are in sorted order.
public static boolean isSorted(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        if (a[i] > a[i + 1]) {
            return false;
        }
    }
    return true;
}
```

Bogo sort code, cont'd.

```
// Shuffles an array of ints by randomly swapping each
// element with an element ahead of it in the array.
public static void shuffle(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // pick a random index in [i+1, a.length-1]
        int range = a.length - 1 - (i + 1) + 1;
        int j = (int) (Math.random() * range + (i + 1));
        swap(a, i, j);
    }
}

// Swaps a[i] with a[j].
public static void swap(int[] a, int i, int j) {
    if (i != j) {
        int temp = a[i];
        a[i] = a[j];
        a[j] = temp;
    }
}
```

Selection sort

- **selection sort**: Orders a list of values by repeatedly putting the smallest or largest unplaced value into its final position.

  The algorithm:
  - Look through the list to find the smallest value.
  - Swap it so that it is at index 0.

  - Look through the list to find the second-smallest value.
  - Swap it so that it is at index 1.

    ...

  - Repeat until all values are in their proper places.

Selection sort example

- Initial array:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- After 1st, 2nd, and 3rd passes:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	**-4**	18	12	**22**	27	30	36	50	7	68	91	56	2	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	**2**	12	22	27	30	36	50	7	68	91	56	**18**	85	42	98	25

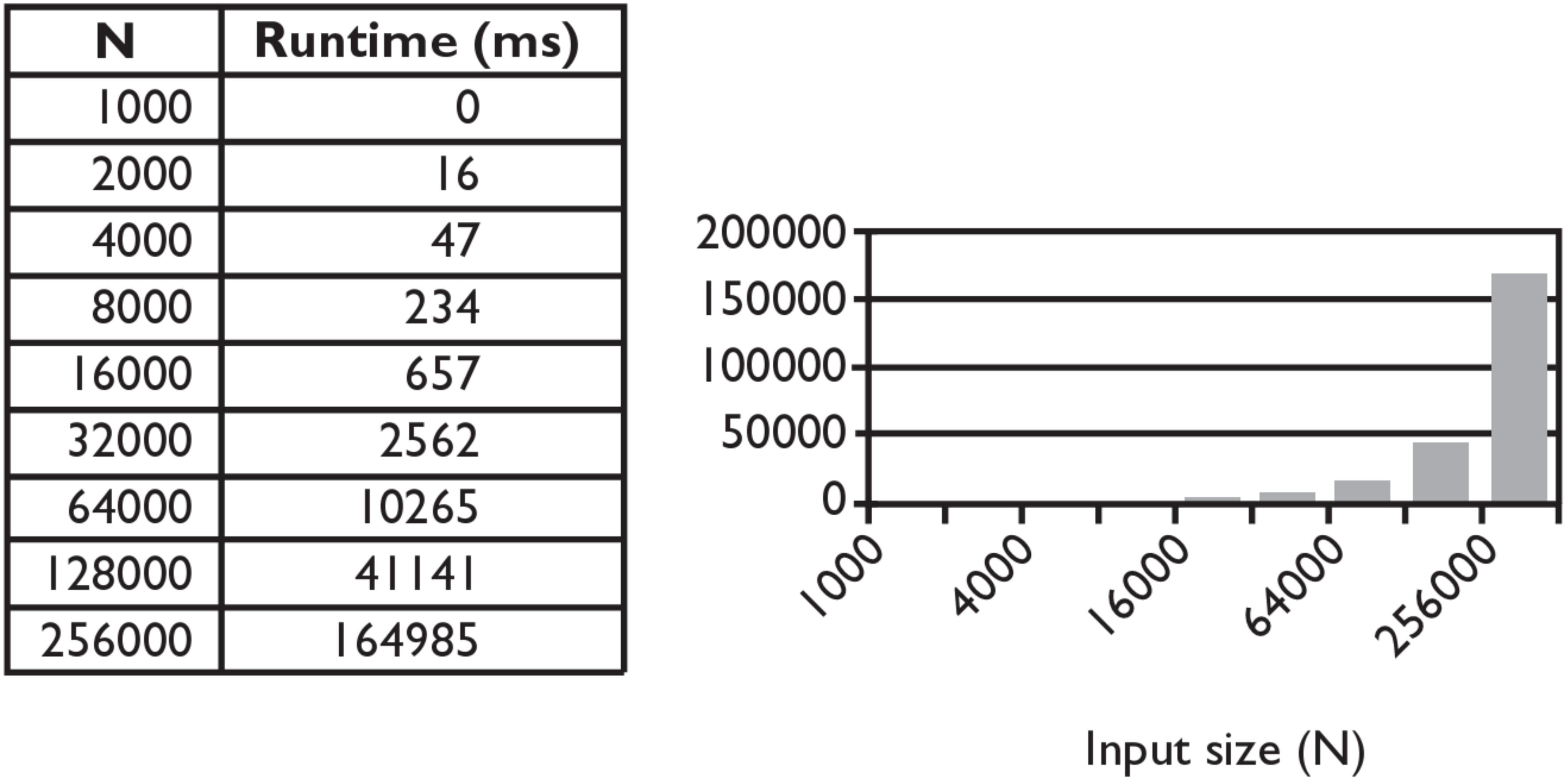
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	**7**	22	27	30	36	50	**12**	68	91	56	18	85	42	98	25

Selection sort code

```
// Rearranges the elements of a into sorted order using
// the selection sort algorithm.
public static void selectionSort(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // find index of smallest remaining value
        int min = i;
        for (int j = i + 1; j < a.length; j++) {
            if (a[j] < a[min]) {
                min = j;
            }
        }
        // swap smallest value its proper place, a[i]
        swap(a, i, min);
    }
}
```

Selection sort runtime (Fig. 13.6)

- What is the complexity class (Big-Oh) of selection sort?



N	Runtime (ms)
1000	0
2000	16
4000	47
8000	234
16000	657
32000	2562
64000	10265
128000	41141
256000	164985

Similar algorithms

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- **bubble sort**: Make repeated passes, swapping adjacent values
  - slower than selection sort (has to do more swaps)

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	18	12	-4	22	27	30	36	7	50	68	56	2	85	42	91	25	98

22 ⟶ 50 ⟶ 91 ⟶ 98 ⟶

- **insertion sort**: Shift each element into a sorted sub-array
  - faster than selection sort (examines fewer values)

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	12	18	22	27	30	36	50	7	68	91	56	2	85	42	98	25

sorted sub-array (indexes 0-7)

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Merge sort

- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.

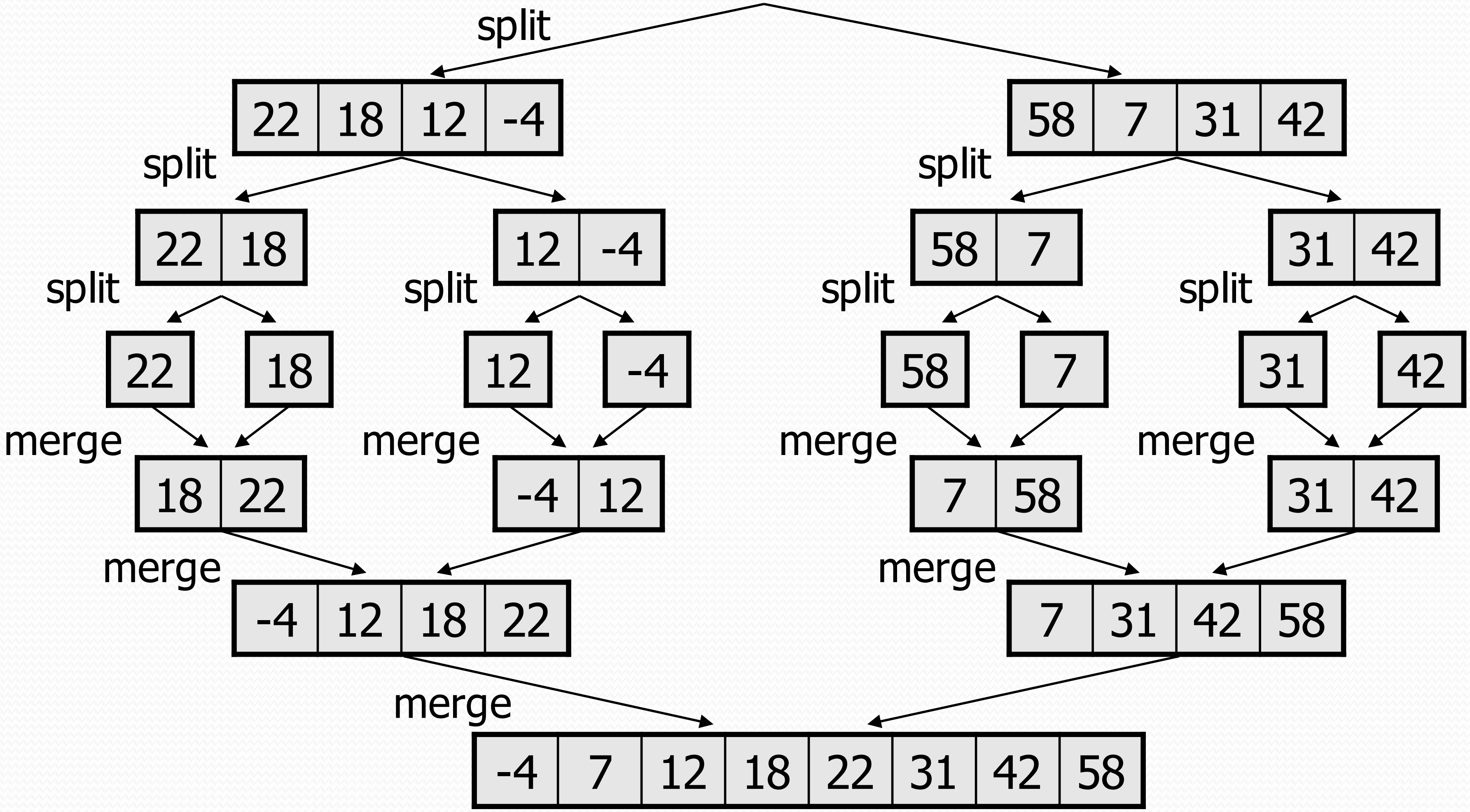
  The algorithm:

  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

Merge sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42



split

22 18 12 -4 | 58 7 31 42

split | split

22 18 | 12 -4 | 58 7 | 31 42

split | split | split | split

22 | 18 | 12 | -4 | 58 | 7 | 31 | 42

merge | merge | merge | merge

18 22 | -4 12 | 7 58 | 31 42

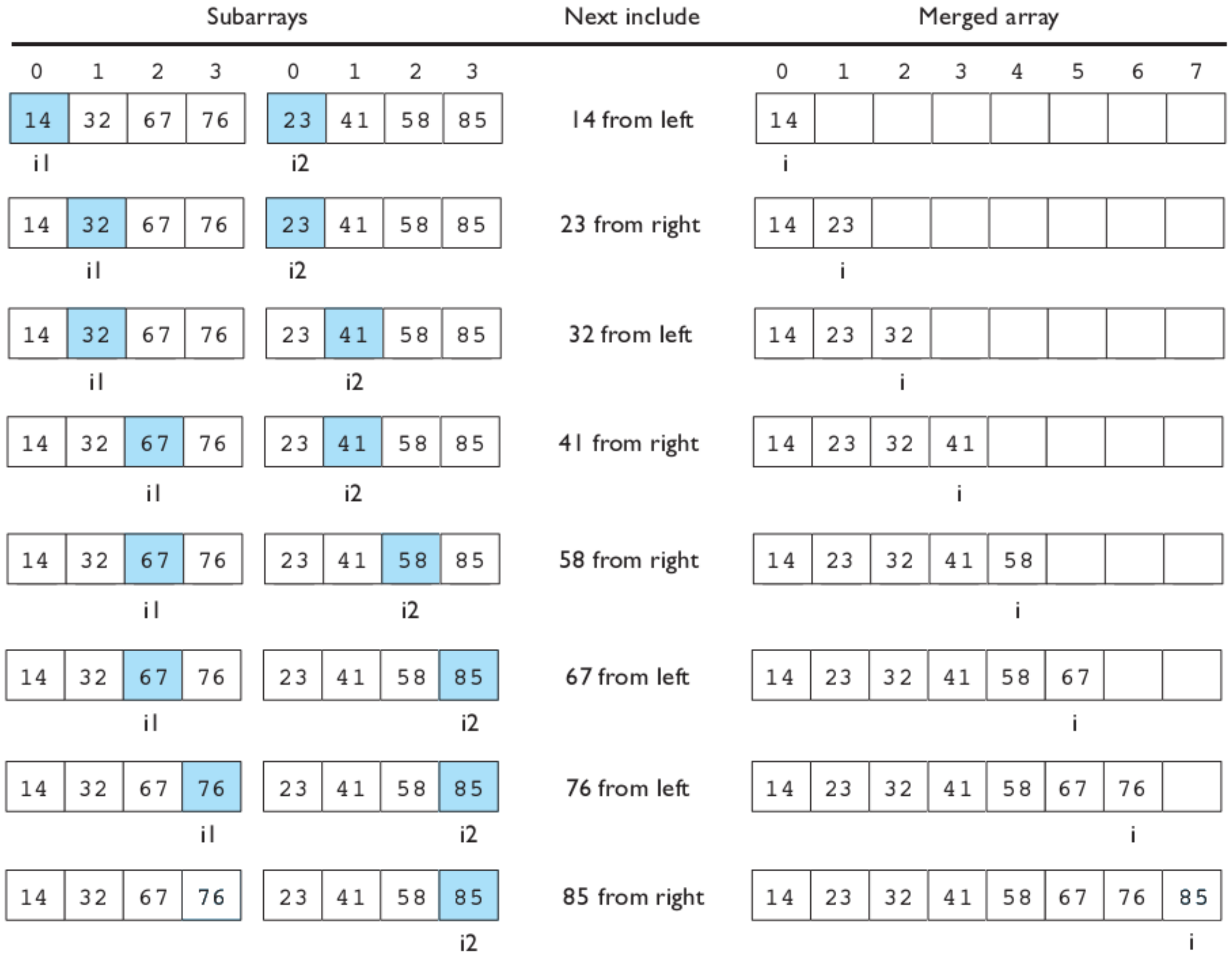
merge | merge

-4 12 18 22 | 7 31 42 58

merge

-4 7 12 18 22 31 42 58

Merging sorted halves



Subarrays								Next include	Merged array							
0	1	2	3	0	1	2	3		0	1	2	3	4	5	6	7
14	32	67	76	23	41	58	85	14 from left	14							
i1				i2					i							
14	32	67	76	23	41	58	85	23 from right	14	23						
	i1			i2						i						
14	32	67	76	23	41	58	85	32 from left	14	23	32					
	i1				i2						i					
14	32	67	76	23	41	58	85	41 from right	14	23	32	41				
		i1			i2							i				
14	32	67	76	23	41	58	85	58 from right	14	23	32	41	58			
		i1				i2							i			
14	32	67	76	23	41	58	85	67 from left	14	23	32	41	58	67		
		i1					i2							i		
14	32	67	76	23	41	58	85	76 from left	14	23	32	41	58	67	76	
			i1				i2								i	
14	32	67	76	23	41	58	85	85 from right	14	23	32	41	58	67	76	85
							i2									i

Merge sort

- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.

  The algorithm:
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

Merge halves code

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
public static void merge(int[] result, int[] left,
                                       int[] right) {
    int i1 = 0;   // index into left array
    int i2 = 0;   // index into right array

    for (int i = 0; i < result.length; i++) {
        if (i2 >= right.length ||
           (i1 < left.length && left[i1] <= right[i2])) {
            result[i] = left[i1];    // take from left
            i1++;
        } else {
            result[i] = right[i2];   // take from right
            i2++;
        }
    }
}
```

Merge sort code

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm.
public static void mergeSort(int[] a) {
    // split array into two halves
    int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
    int[] right = Arrays.copyOfRange(a, a.length/2,
  a.length);

    // sort the two halves
    ...

    // merge the sorted halves into a sorted whole
    merge(a, left, right);
}
```

Merge sort code 2

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm (recursive).
public static void mergeSort(int[] a) {
    if (a.length >= 2) {
        // split array into two halves
        int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
        int[] right = Arrays.copyOfRange(a, a.length/2,
  a.length);

        // sort the two halves
        mergeSort(left);
        mergeSort(right);

        // merge the sorted halves into a sorted whole
        merge(a, left, right);
    }
}
```

Merge sort runtime

- What is the complexity class (Big-Oh) of merge sort?

N	Runtime (ms)
1000	0
2000	0
4000	0
8000	0
16000	0
32000	15
64000	16
128000	47
256000	125
512000	250
1e6	532
2e6	1078
4e6	2265
8e6	4781
1.6e7	9828
3.3e7	20422
6.5e7	42406
1.3e8	88344