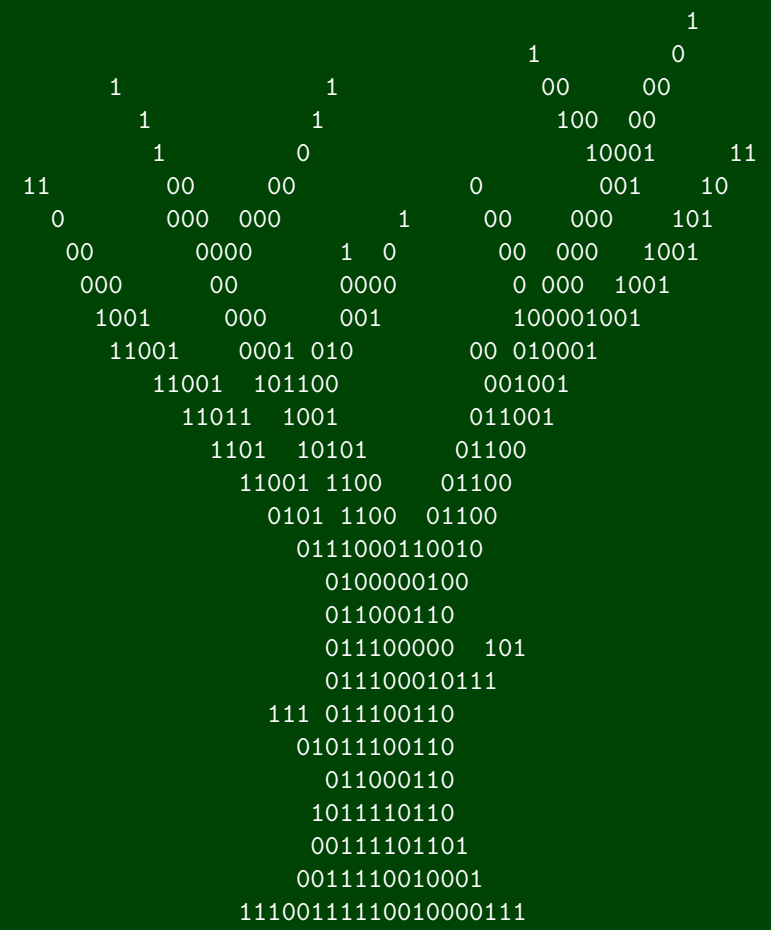
CSE 143

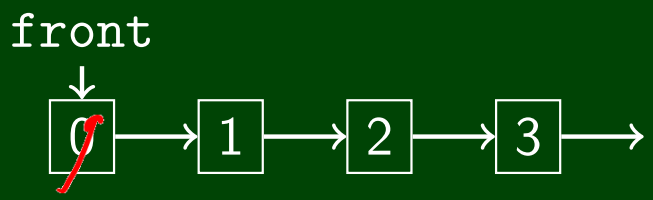
Computer Programming II

Binary Trees



Outline

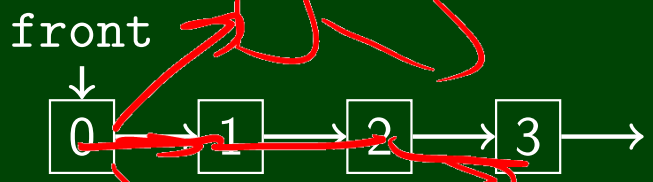
Consider the following standard LinkedList:



Recall the definition of a ListNode

```
1 public class Node {
2    public int data;
3    public Node next;
4
5    public Node(int data, Node next) {
6       this.data = data;
7       this.next = next;
8    }
9 }
```

Consider the following standard LinkedList:



Recall the definition of a ListNode

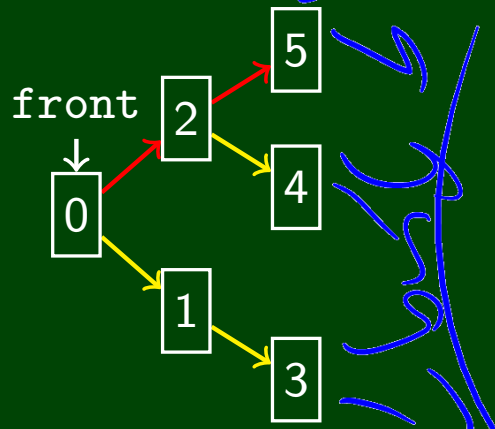
```
1 public class Node {
2    public int data;
3    public Node next;
4
5    public Node(int data, Node next) {
6       this.data = data;
7       this.next = next;
8    }
9 }
```

What if we added more fields?

- Multiple data fields?
- Multiple "next" fields?

Nodes with Multiple next Fields

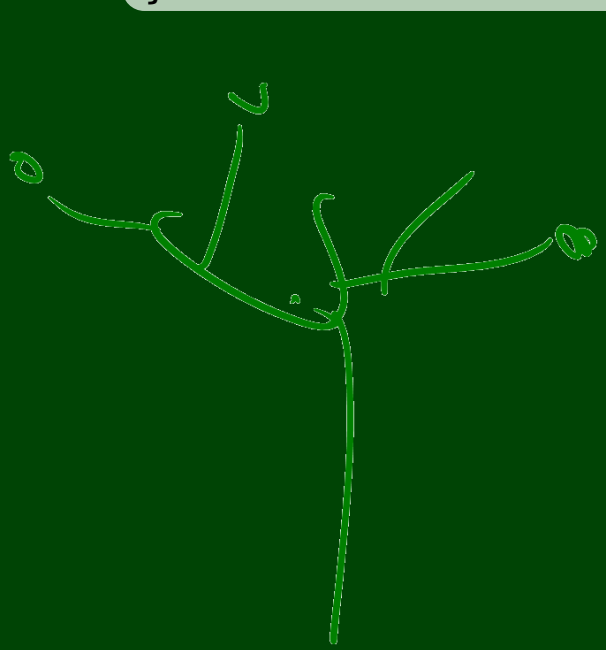
```
1  public class Node {
2     public int data;
3     public Node next1;
4     public Node next2;
5
6     public Node(int data, Node next1, Node next2) {
7        this.data = data;
8        this.next1 = next1;
9        this.next2 = next2;
10    }
11 }
```

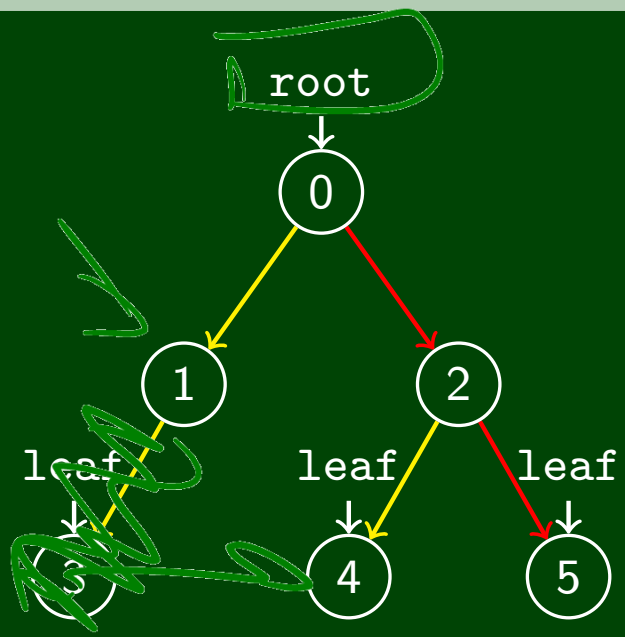


(yellow is next2; red is next1)

Binary Trees

```
1  public class Node {
2     public int data;
3     public Node left;
4     public Node right;
5
6     public Node(int data, Node left, Node right) {
7        this.data = data;
8        this.left = left;
9        this.right = right;
10    }
11 }
```

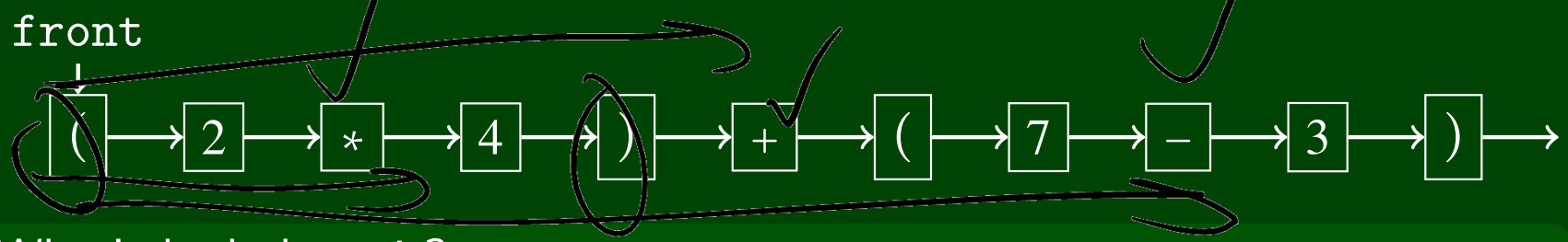




(red is right; yellow is left)

Back To `LinkedLists`

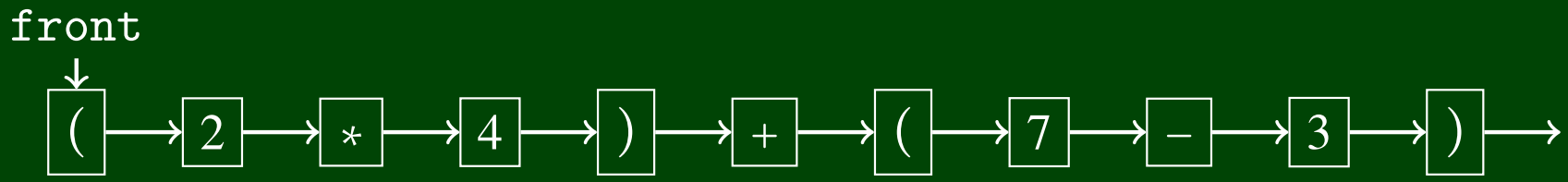
Consider the following `LinkedList` of a mathematical expression:



What's bad about it?

Back To `LinkedLists`

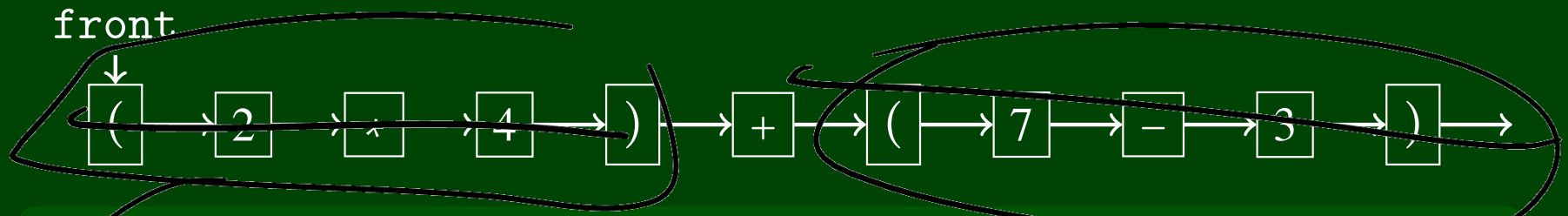
Consider the following `LinkedList` of a mathematical expression:



What's bad about it?

- It doesn't really help us with the structure
- Looking at it doesn't really show us what's going on

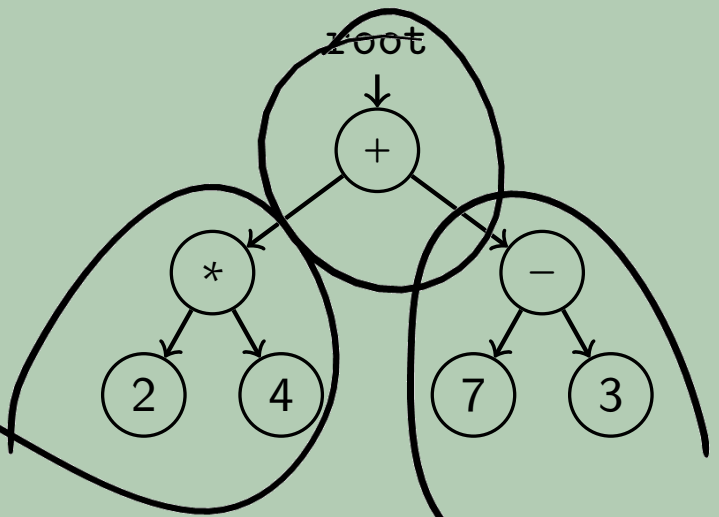
Consider the following LinkedList of a mathematical expression:



What's bad about it?

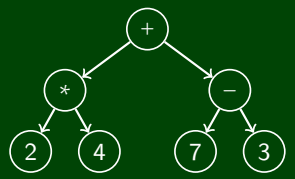
- It doesn't really help us with the structure
- Looking at it doesn't really show us what's going on

What about this structure instead?



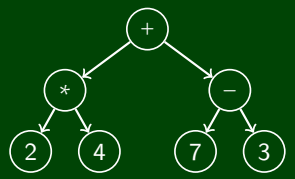
Now we can see the order of operations much more clearly!

- Parsing (Programming Languages, Math, etc.)

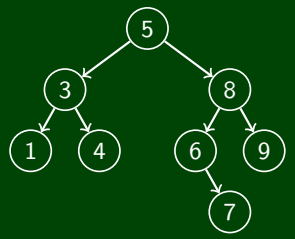


Uses of Trees

- Parsing (Programming Languages, Math, etc.)

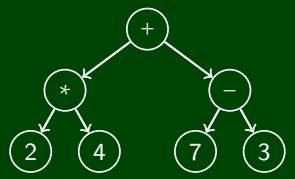


- Implementing `TreeSet`

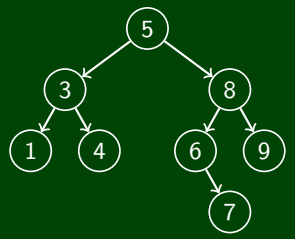


Uses of Trees

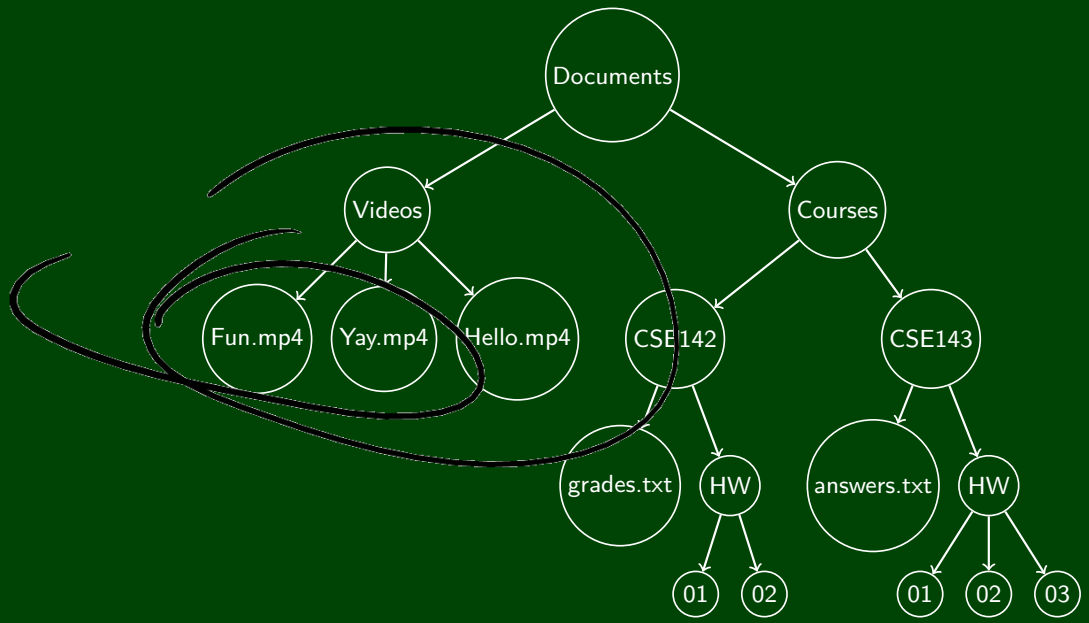
- Parsing (Programming Languages, Math, etc.)



- Implementing `TreeSet`

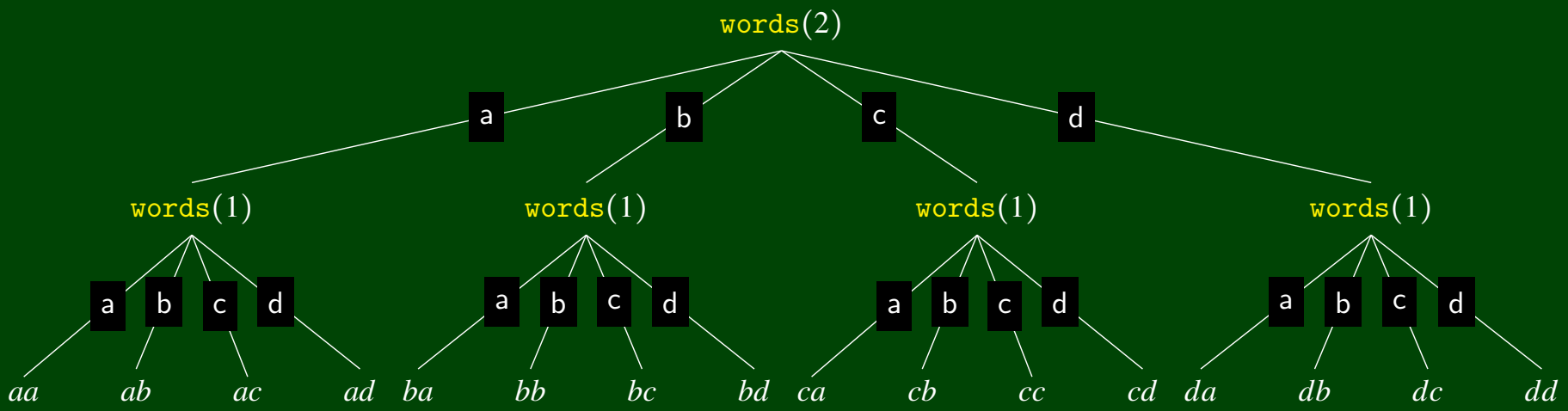


- Directory File Structure



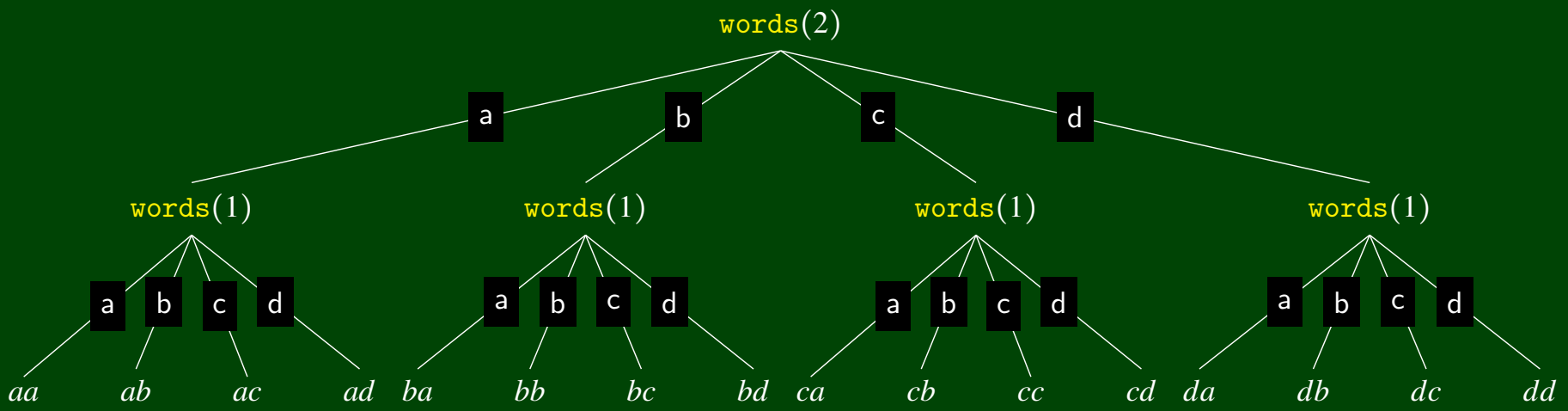
More Uses of Trees

- Recursive Trees (including things like games of Tic-Tac-Toe)



More Uses of Trees

- Recursive Trees (including things like games of Tic-Tac-Toe)



- Compression (this will be your last assignment!)

Printing A `LinkedList` (Again)


```
1 public void print() {
2     Node current = this.front;
3     while (current != null) {
4         System.out.print(current.data + " ");
5         current = current.next;
6     }
7 }
```

public class LinkedList{
private ListNode front;

We'd like to figure out how to print trees. Since `LinkedLists` are "simpler versions of trees", they might help.

Public void print(){

Public void print(LN current)
Print current.data
print "the rest"
current.next
(recursively)

Printing A `LinkedList` (Again)


```
1 public void print() {
2    Node current = this.front;
3    while (current != null) {
4       System.out.print(current.data + " ");
5       current = current.next;
6    }
7 }
```

We'd like to figure out how to print trees. Since `LinkedLists` are "simpler versions of trees", they might help.

How do we go in every direction in a tree?

USE RECURSION!

Printing a `LinkedList` Recursively

To print a `LinkedList`...

- Print the **front** of the list
- Print the **next** of the list (recursively)

Code

```
1  public void print() {
2     print(this.front);
3  }
4
5  public void print(Node c) {
6     if (c != null) {
7        System.out.print(c.data + " ");
8        print(c.next);
9     }
10 }
```

Printing a `Tree` Recursively

To print a `BinaryTree`...

- Print the **root** of the tree
- Print the **left** of the tree (recursively)
- Print the **right** of the tree (recursively)

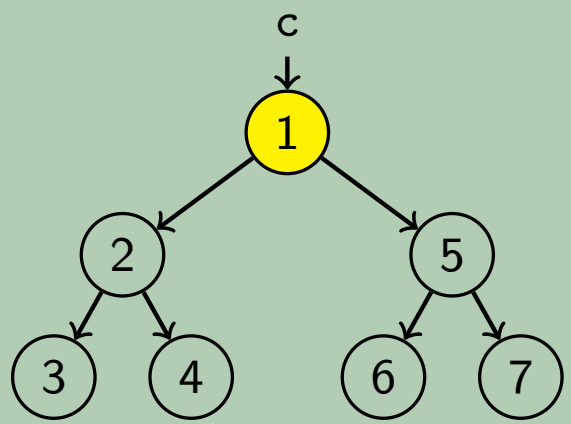
Code

```
1  public void print() {
2     print(this.root);
3  }
4
5  public void print(Node c) {
6     if (c != null) {
7        System.out.print(c.data + " ");
8        print(c.left);
9        print(c.right);
10    }
11 }
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



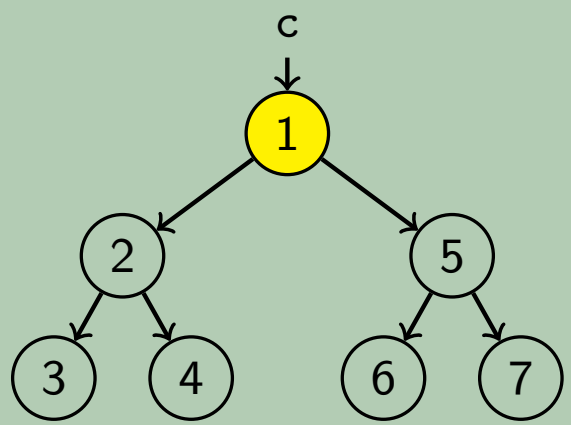
OUTPUT

```
>> 1
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

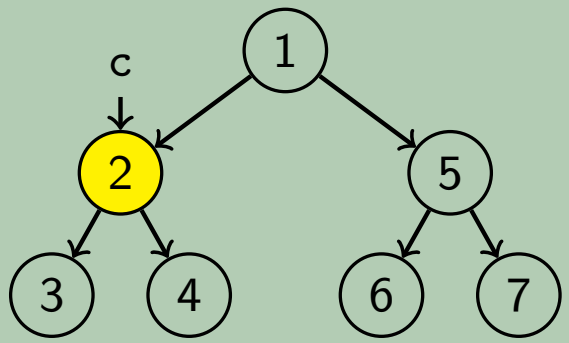
```
>> 1
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2
```

Printing a Tree Example

```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
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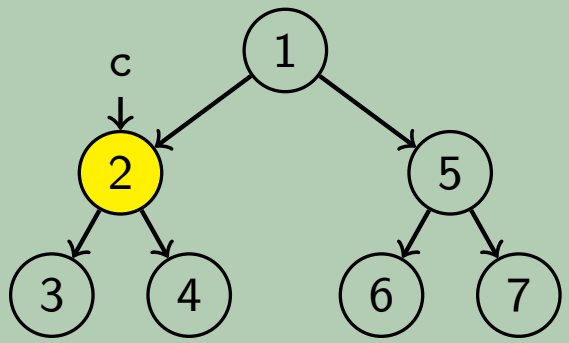

OUTPUT

```
>> 1 2
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
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4
5
6
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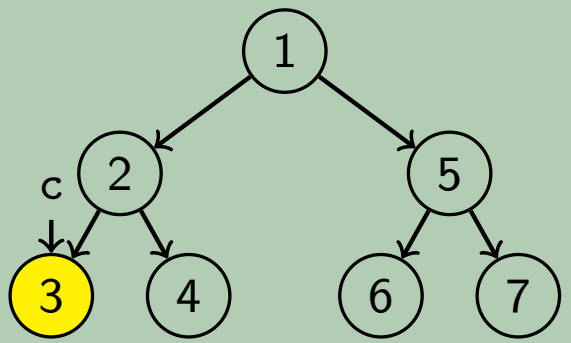
Trace



OUTPUT

```
>> 1 2 3
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
3
4
5
6
7  }
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```
1  public void print(Node c) { // c = ③
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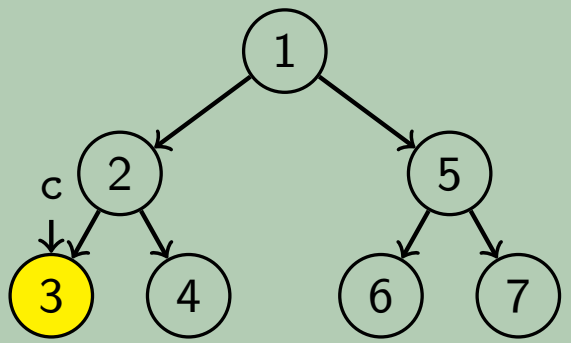

OUTPUT

```
>> 1 2 3
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
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4
5
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```

```
1  public void print(Node c) { // c = ③
2     if (c != null) {
3
4
5
6
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```
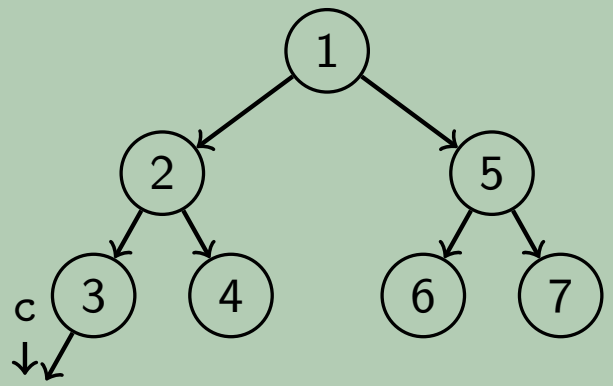

```
1  public void print(Node c) { // c = null
2     if (c != null) {              // c is null!
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
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1  public void print(Node c) { // c = ②
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```
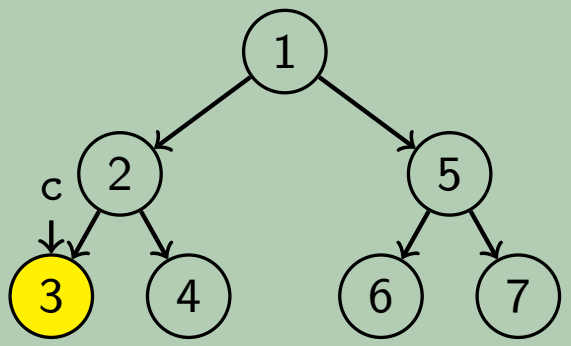

Trace



OUTPUT

```
>> 1 2 3
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
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3        System.out.print(c.data + " ");
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```

```
1  public void print(Node c) { // c = null
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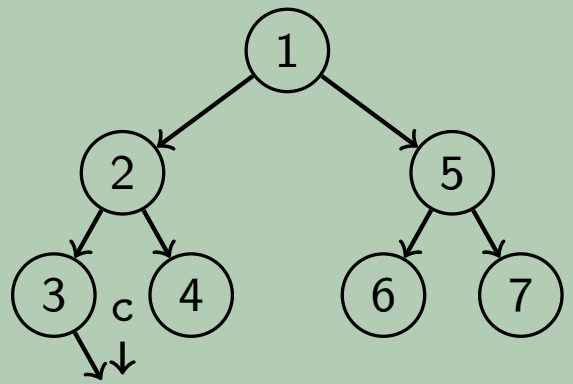
Trace



OUTPUT

```
>> 1 2 3
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
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4
5
6
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1  public void print(Node c) { // c = ③
2     if (c != null) {
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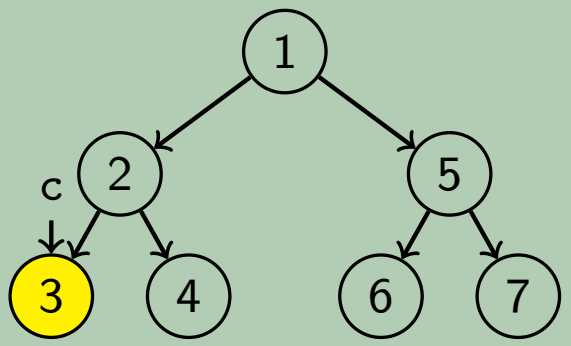


OUTPUT

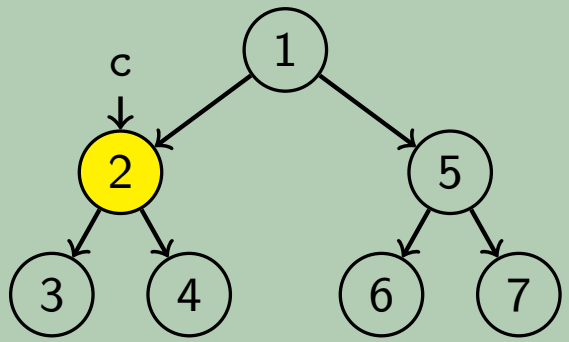
```
>> 1 2 3
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3
```

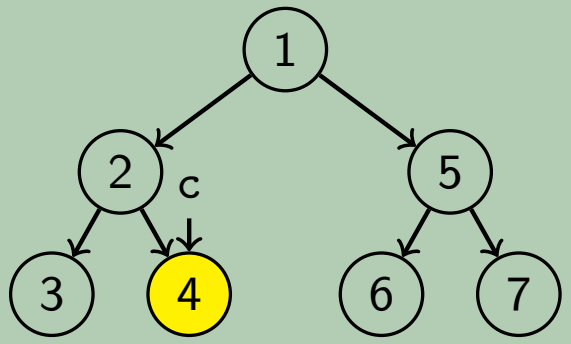
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ④
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

```
1  public void print(Node c) { // c = ②
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3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```
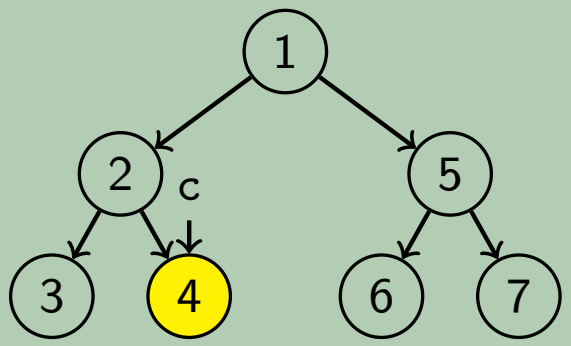

```
1  public void print(Node c) { // c = ④
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4
```

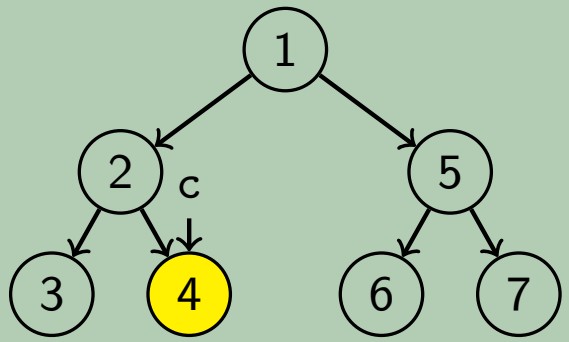
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ④
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4
```

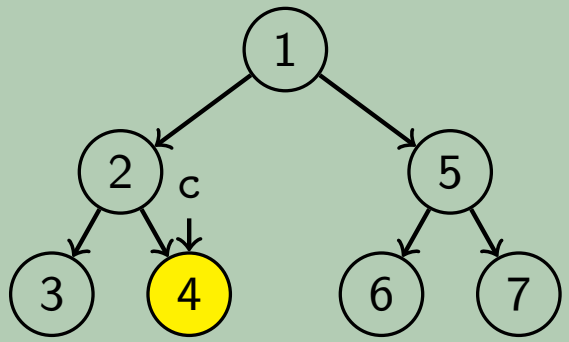
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ④
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

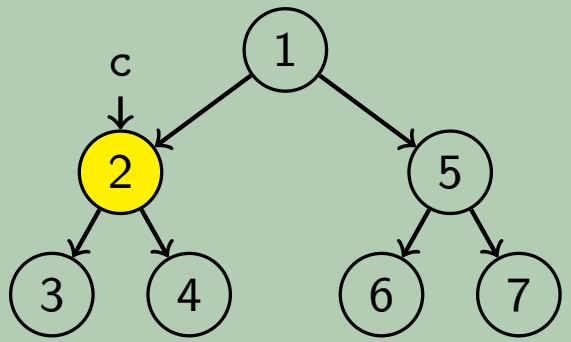
```
>> 1 2 3 4
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ②
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



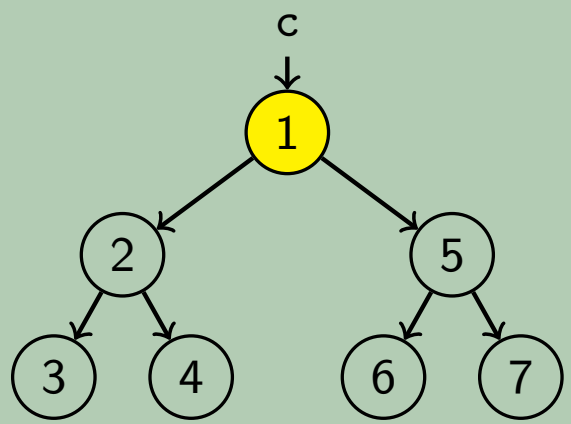
OUTPUT

```
>> 1 2 3 4
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

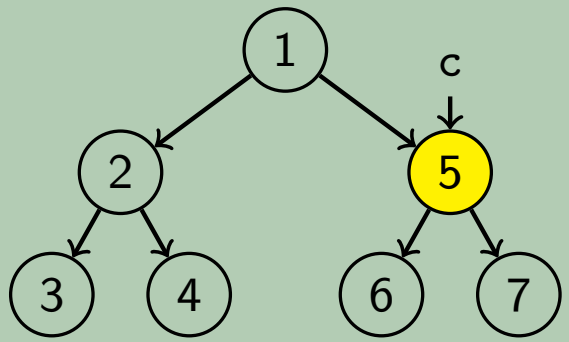
```
>> 1 2 3 4
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

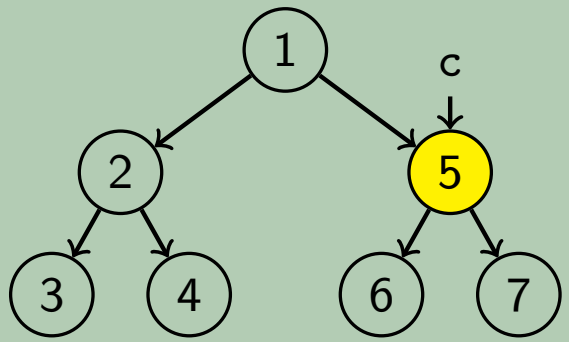
```
>> 1 2 3 4 5
```

Printing a Tree Example


```
1  public void print(Node c) { // c = (1)
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = (5)
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5
```

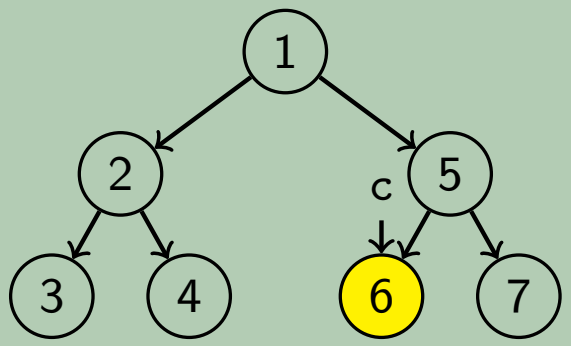
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

```
1  public void print(Node c) { // c = ⑥
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6
```

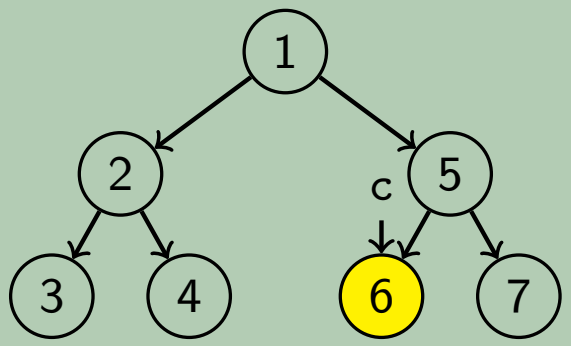
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑥
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6
```

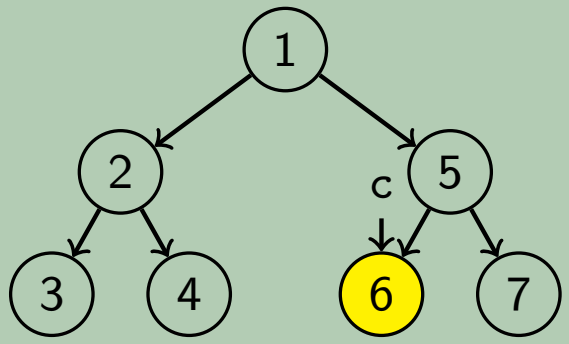
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑥
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6
```

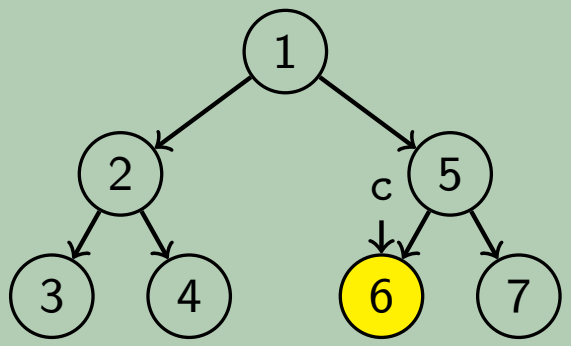
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑥
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

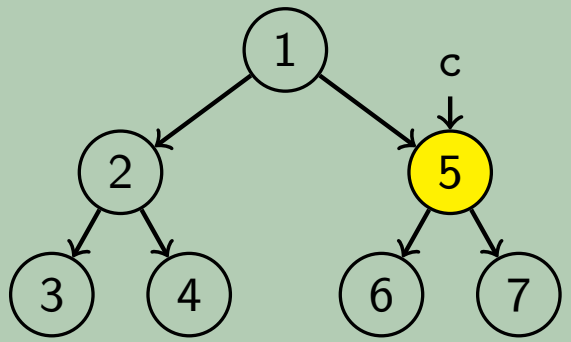
```
>> 1 2 3 4 5 6
```

Printing a Tree Example

```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6
```

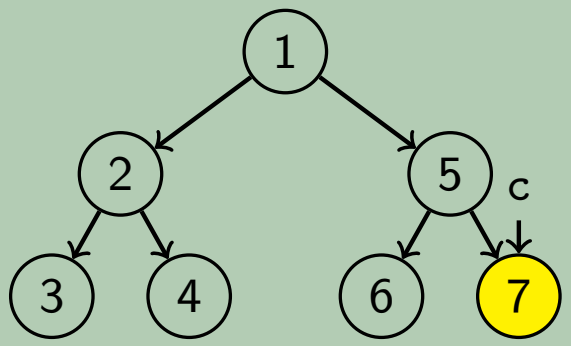
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑦
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6 7
```

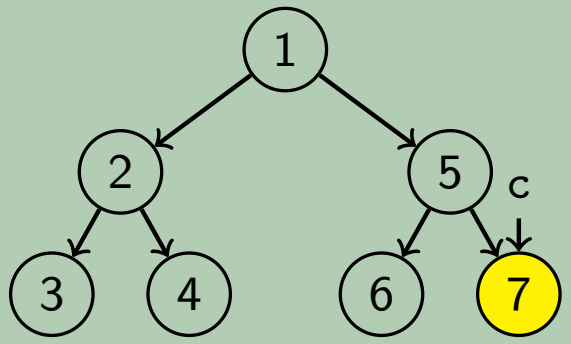
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

```
1  public void print(Node c) { // c = ⑦
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6 7
```

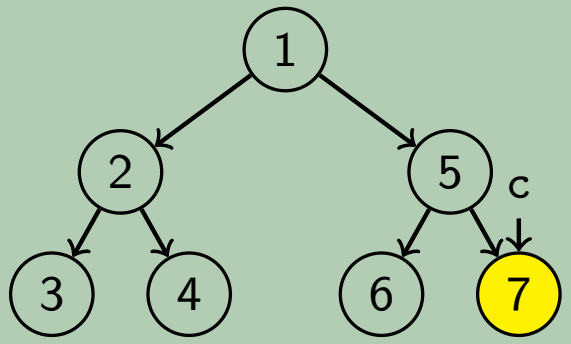
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑦
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6 7
```

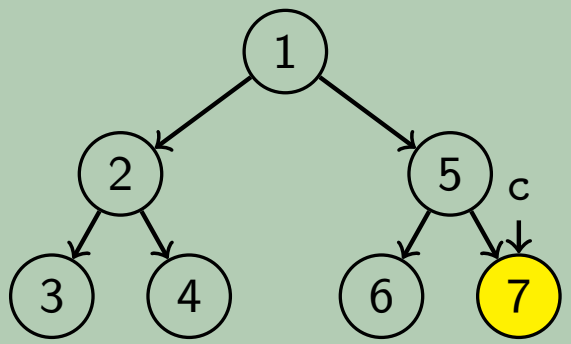
Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

```
1  public void print(Node c) { // c = ⑦
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

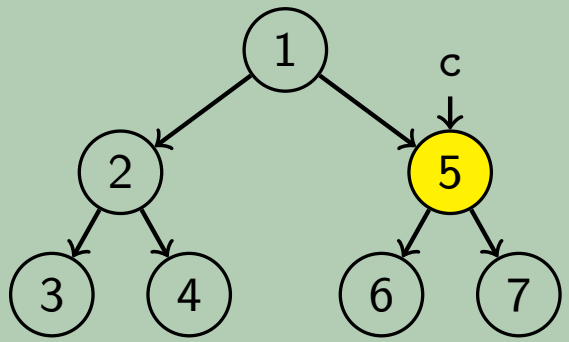
```
>> 1 2 3 4 5 6 7
```

Printing a Tree Example


```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3
4
5
6
7  }
```

```
1  public void print(Node c) { // c = ⑤
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



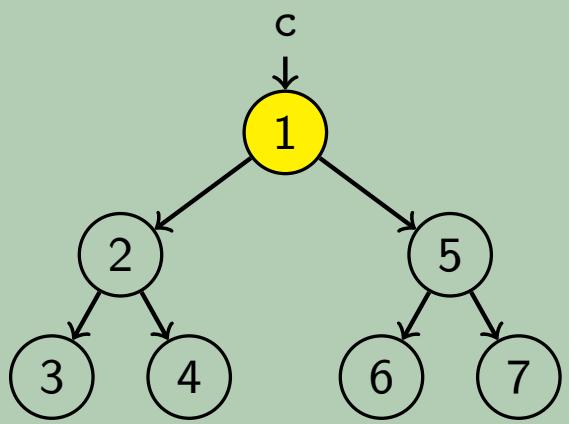
OUTPUT

```
>> 1 2 3 4 5 6 7
```

Printing a Tree Example

```
1  public void print(Node c) { // c = ①
2     if (c != null) {
3        System.out.print(c.data + " ");
4        print(c.left);
5        print(c.right);
6     }
7  }
```

Trace



OUTPUT

```
>> 1 2 3 4 5 6 7
```

Pre-Order Traversal

```
1 public void print(Node c) {
2    if (c != null) {
3       System.out.print(c.data + " "); // print
4       print(c.left);                  // left
5       print(c.right);                 // right
6    }
7 }
```

In-Order Traversal

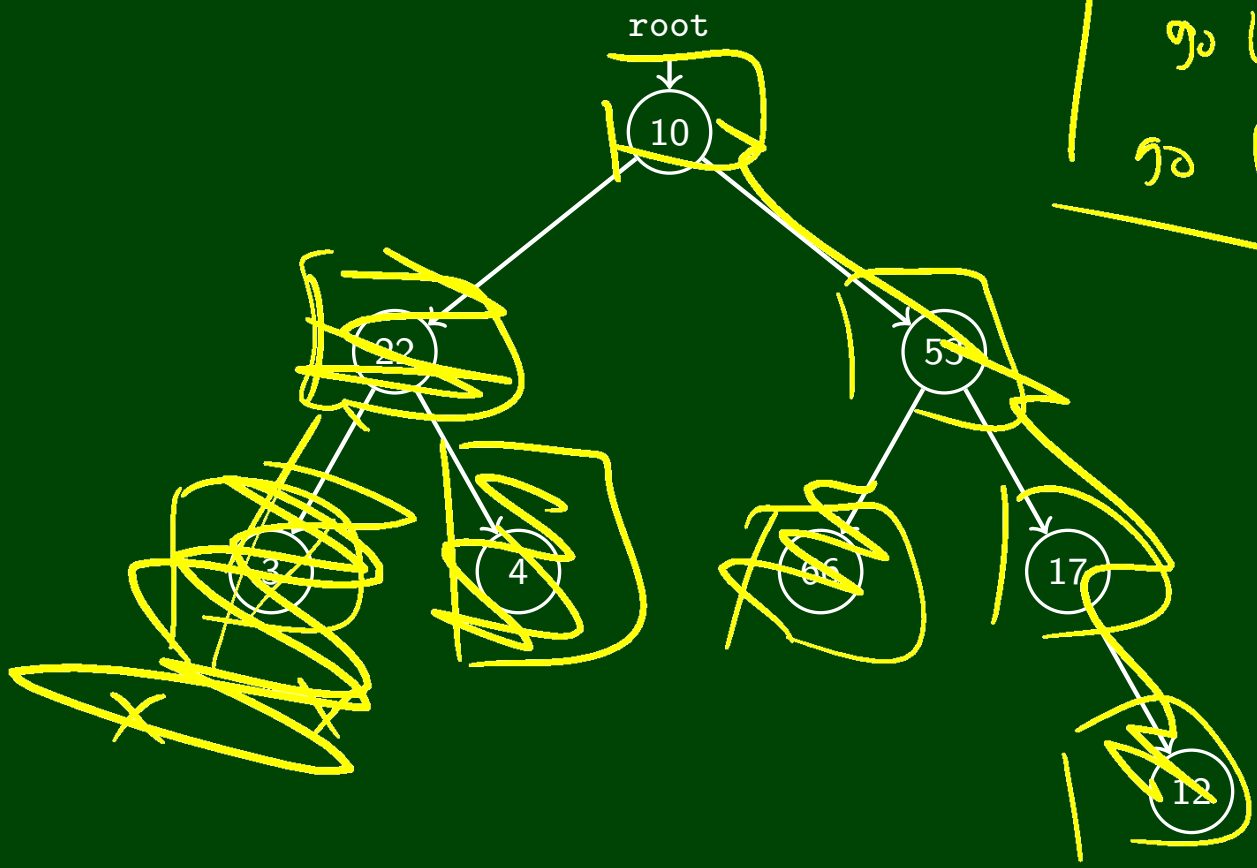
```
1 public void print(Node c) {
2    if (c != null) {
3       print(c.left);                  // left
4       System.out.print(c.data + " "); // print
5       print(c.right);                 // right
6    }
7 }
```

Post-Order Traversal

```
1 public void print(Node c) {
2    if (c != null) {
3       print(c.left);                  // left
4       print(c.right);                 // right
5       System.out.print(c.data + " "); // print
6    }
7 }
```

Tree Traversal Example

Consider the following binary tree:



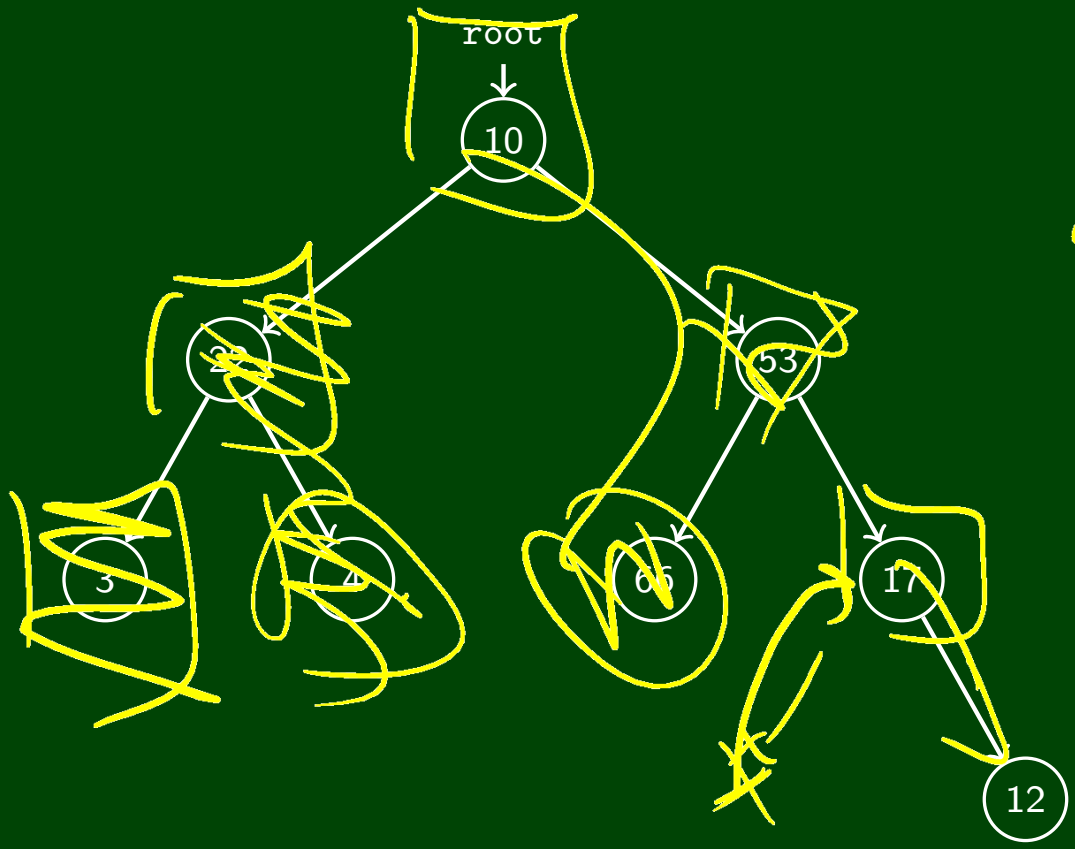
Print
go left
go right

Compute the Pre-Order, In-Order, and Post-Order Traversals:

- Pre-Order: 10 22 3 4 53 66 17 12

Tree Traversal Example

Consider the following binary tree:

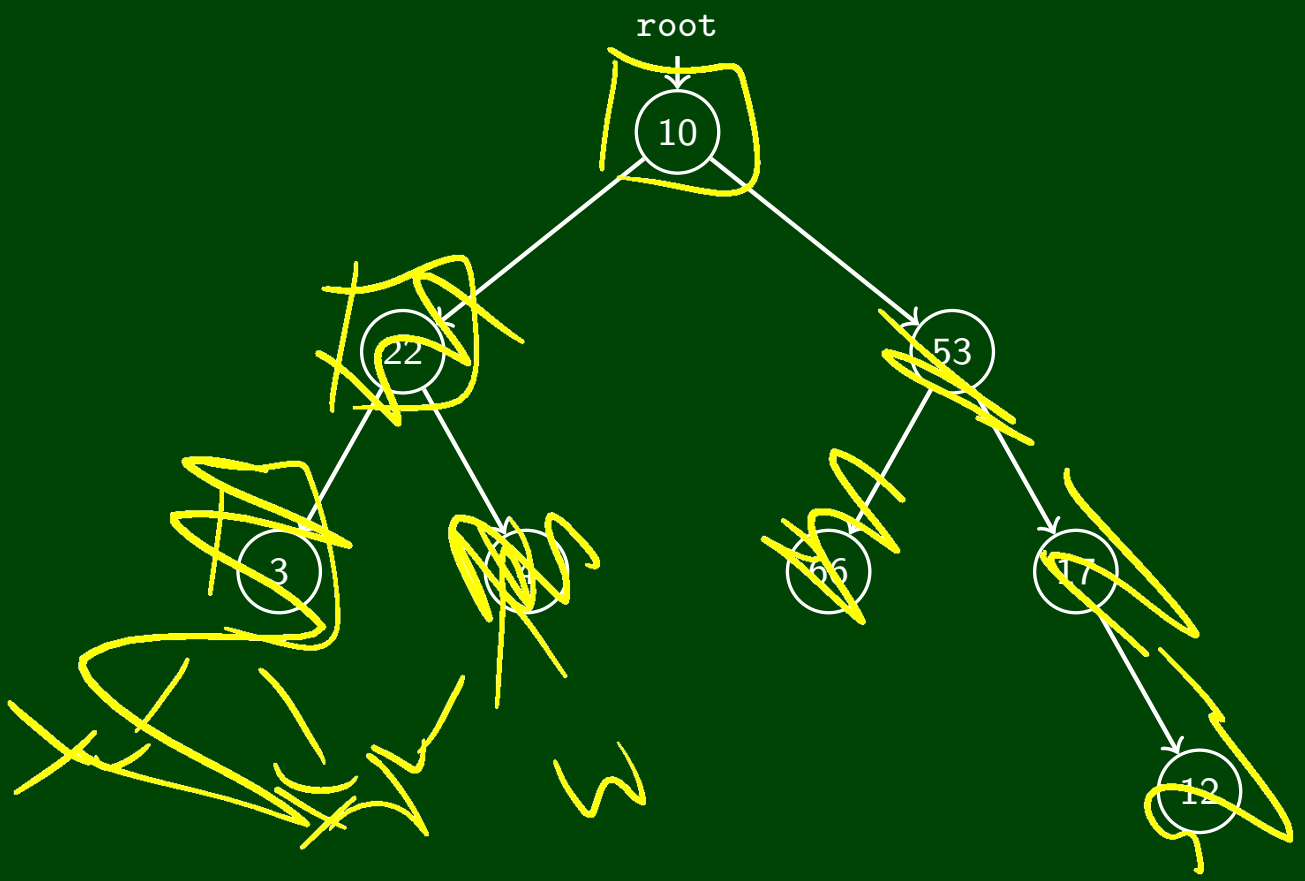


Compute the Pre-Order, In-Order, and Post-Order Traversals:

- Pre-Order: **10, 22, 3, 4, 53, 66, 17, 12**
- In-Order:

Tree Traversal Example

Consider the following binary tree:

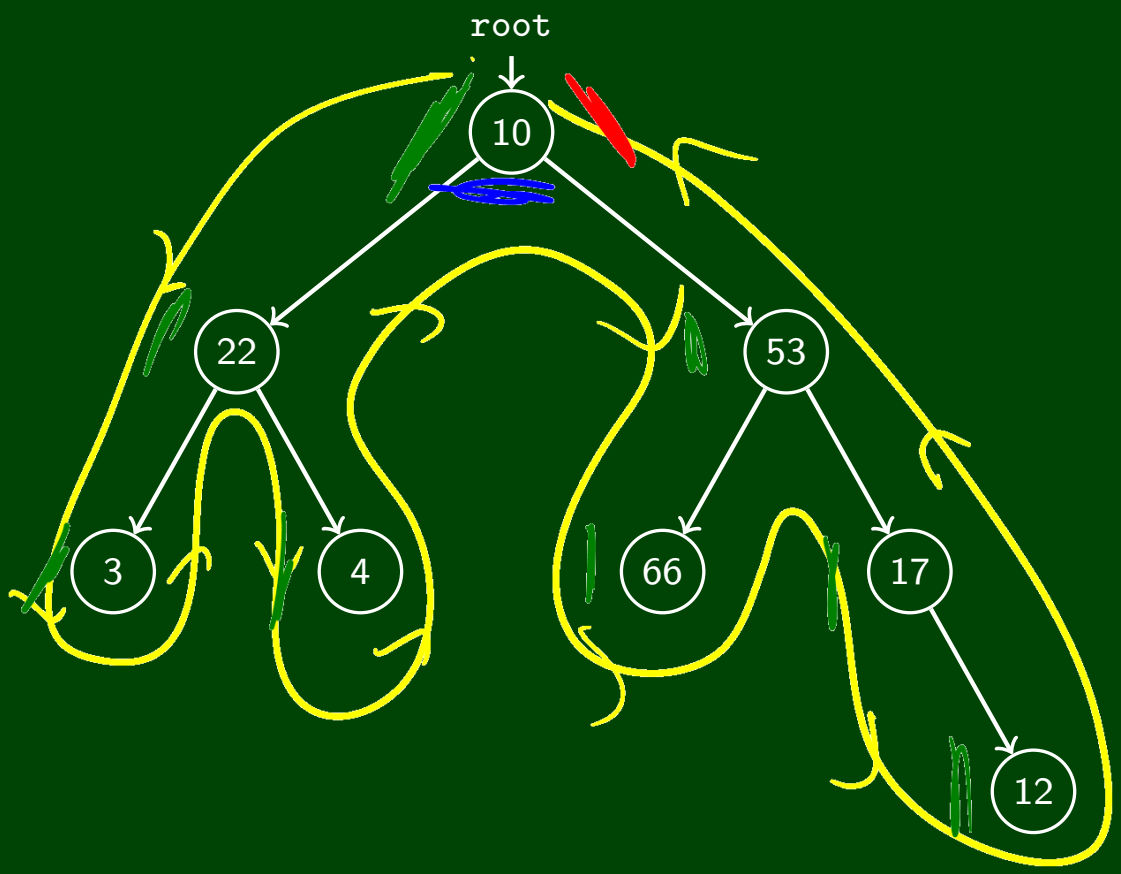


Compute the Pre-Order, In-Order, and Post-Order Traversals:

- Pre-Order: **10, 22, 3, 4, 53, 66, 17, 12**
- In-Order: **3, 22, 4, 10, 66, 53, 17, 12**
- Post-Order:

Tree Traversal Example

Consider the following binary tree:

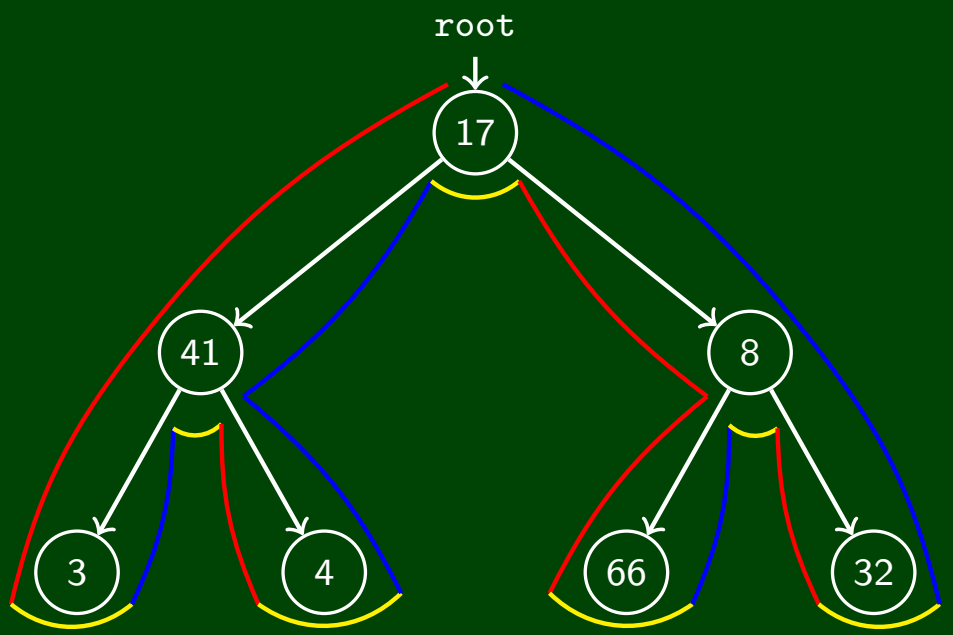


Compute the Pre-Order, In-Order, and Post-Order Traversals:

- Pre-Order: **10, 22, 3, 4, 53, 66, 17, 12**
- In-Order: **3, 22, 4, 10, 66, 53, 17, 12**
- Post-Order: **3, 4, 22, 66, 12, 17, 53, 10**

To Quickly Generate A Traversal

- Trace a path around the tree
- As you pass a node on the proper side, process it:
  - Pre-Order: left
  - In-Order: bottom
  - Post-Order: right



Binary Tree methods are just normal recursive functions. The base case/recursive calls will always be similar.

Writing a Binary Tree Method

- The base case is `current == null`.
- First recursive case is `method(current.left)`
- Second recursive case is `method(current.right)`

```
1  public type method(...) {
2    return method(this.root, ...);
3  }
4  private type method(TreeNode current, ...) {
5    if (current == null) { /* DO BASE CASE */ }
6
7    // Do the left recursive case:
8    type leftResult = method(current.left, ...);
9
10   // Do the right recursive case:
11   type rightResult = method(current.right, ...);
12
13   /* Use the left and right results... */
14   return ...;
15 }
```

`contains()`

Write a method, in the `IntTree` class, called `contains()`:

```
public boolean contains(int value);
```

that returns true if the tree contains `value` and false otherwise.

contains()

Write a method, in the IntTree class, called contains():

```
public boolean contains(int value);
```

that returns true if the tree contains value and false otherwise.

```
1  public boolean contains(int value) {
2     return contains(this.root, value);
3  }
4  private boolean contains(IntTreeNode current, int value) {
5     /* If the tree is null, it definitely doesn't contain value... */
6     if (current == null) { return false; }
7
8     /* If current *is* value, we found it! */
9     else if (current.data == value) { return true; }
10
11    else {
12       boolean leftContainsValue = contains(current.left, value);
13       boolean rightContainsValue = contains(current.right, value);
14       return leftContainsValue || rightContainsValue;
15    }
16 }
```

Some Tree Tips!

- Trees are just generalized `LinkedLists`. So, all of the things you learned about references with `LinkedLists` are going to apply to trees as well
- Almost all the tree methods you write will be recursive (and will have a private helper that takes in the root)
- Make sure you understand all the traversals; the trick can be very useful.