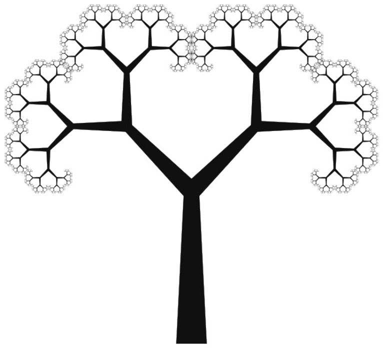
CSE 143

Computer Programming II

Binary Search Trees (BSTs)



Outline

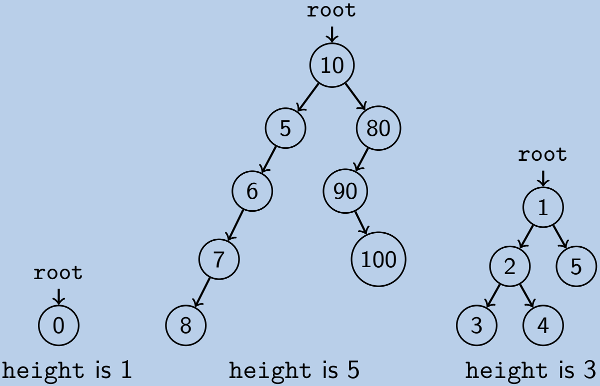
1. More Tree Methods
2. Introducing BSTs
3. BST Methods

height

Write a tree method called `height` (inside the `IntTree` class) with the following method signature:

```
public int height()
```

that returns the number of nodes on the **longest path** from the root to any leaf. For example,



height Solution

```
public int height() {
   return height(this.root);
}

private int height(IntTreeNode current) {
   // A null tree has height 0
   if (current == null) {
      return 0;
   }
   else {
      // Find the largest path by taking the max
      // of both branches recursively (and adding 1 for this node)
      return 1 + Math.max(
         height(current.left),
         height(current.right)
      );
   }
}
```

Back to `contains`

Recall `contains()`

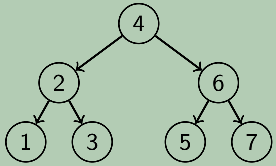
```java
private boolean contains(IntTreeNode current, int value) {
   /* If the tree is null, it definitely doesn't contain value... */
   if (current == null) { return false; }

   /* If current *is* value, we found it! */
   else if (current.data == value) { return true; }

   else {
      return contains(current.left, value) ||
               contains(current.right, value);
   }
}
```

Runtime of `contains(7)`

Consider the following tree:



Recall contains()

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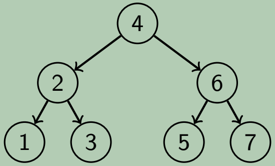
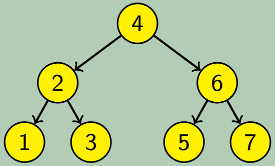


Which nodes do we visit for contains(**7**)



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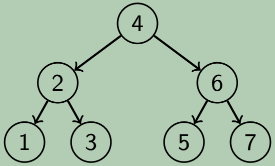

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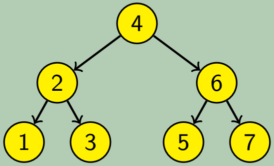


That makes the code $\mathcal{O}(n)$. Can we do better?

Doing Better!

In general, **we can't do better**. BUT, sometimes, we can!

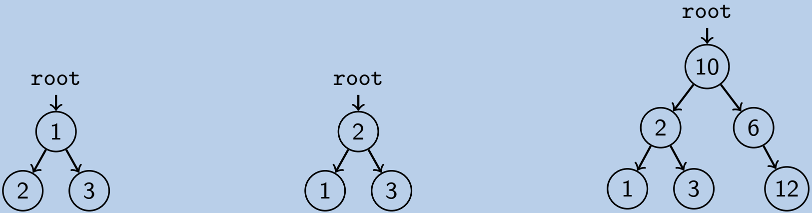
Definition (Binary SEARCH Tree (BST))

A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

To put it another way, a binary tree is a **BST** when:

- All data "to the left of" a node is less than it
- All data "to the right of" a node is greater than it
- All sub-trees of the binary tree are also BSTs

Example (Which of the following are BSTs?)



NO YES NO

Write contains() for a BST

Fix contains so that it takes advantage of the BST properties.

Recall contains()

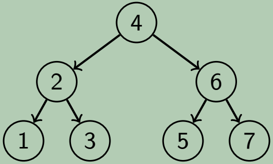
```
private boolean contains(IntTreeNode current, int value) {
   /* If the tree is null, it definitely doesn't contain value... */
   if (current == null) { return false; }

   /* If current *is* value, we found it! */
   else if (current.data == value) { return true; }

   else if (current.data < value) {
      return contains(current.right, value);
   }
   else {
      return contains(current.left, value);
   }
}
```

Runtime of (better) `contains(7)`

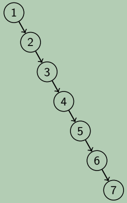
Consider the following tree:



That makes the code $\log n$. Much better!

WARNING!

Consider the following tree:

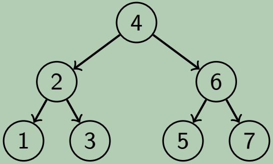


This is the same tree, but now **we have to visit all the nodes!**

Tracing the new `contains`

Runtime of (better) `contains(7)`

Consider the following tree:



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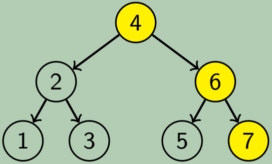

That makes the code $\log n$. Much better!

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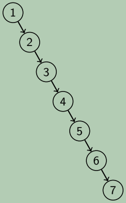
Consider the following tree:



This is the same tree, but now **we have to visit all the nodes!**

add

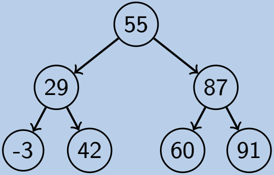
Write a method add in the BST class with the following signature:

```
public void add(int value);
```

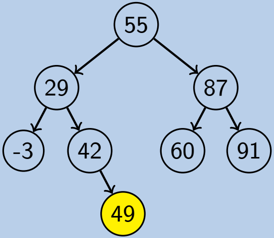
that preserves the BST property.

Example (`tree.add(49)`)

Before



After



Adding to a BST (Attempt #1)

Attempt #1

```
public void add(int value) {
   add(this.root, value);
}
private void add(IntTreeNode current, int value) {
   if (current == null) {
      current = new IntTreeNode(value);
   }
   else if (current.data > value) {
      add(current.left, value);
   }
   else if (current.data < value) {
      add(current.right, value);
   }
}
```

What's wrong with this solution?

Just like with `LinkedLists` where we must change `front` or `.next`, we're not actually changing anything here. We're discarding the result.

`x = change(x)`

Consider the following code:

```
public static void main(String[] args) {
   String s = "hello world";
   s.toUpperCase();
   System.out.println(s);
}
```

OUTPUT

```
>> hello world
```

```
public static void main(String[] args) {
   String s = "hello world";
   s = s.toUpperCase();
   System.out.println(s);
}
```

OUTPUT

```
>> HELLO WORLD
```

We must `USE` the result; otherwise, it gets discarded

x = change(x)

If you want to write a method that can change the object that a variable refers to, you must do three things:

1. Pass in the original state of the object to the method
2. Return the new (possibly changed) object from the method
3. Re-assign the caller's variable to store the returned result

```
p = change(p); // in main
public static Point change(Point thePoint) {
   thePoint = new Point(99, -1);
   return thePoint;
}
```

Adding to a BST (Fixed)

Fixed Attempt

```java
public void add(int value) {
   this.root = add(this.root, value);
}
private IntTreeNode add(IntTreeNode current, int value) {
   if (current == null) {
      current = new IntTreeNode(value);
   }
   else if (current.data > value) {
      current.left = add(current.left, value);
   }
   else if (current.data < value) {
      current.right = add(current.right, value);
   }
   return current;
}
```

This works because we **always update the result**, **always return the result**, and **always update the root**.

BST Tips!

- BSTs can make searching/inserting/etc. much faster.

- Make sure that you can figure out if a tree is a BST or not.

- Whenever you are writing a BST method, you **must** use the `x = change(x)` pattern. It won't work otherwise.