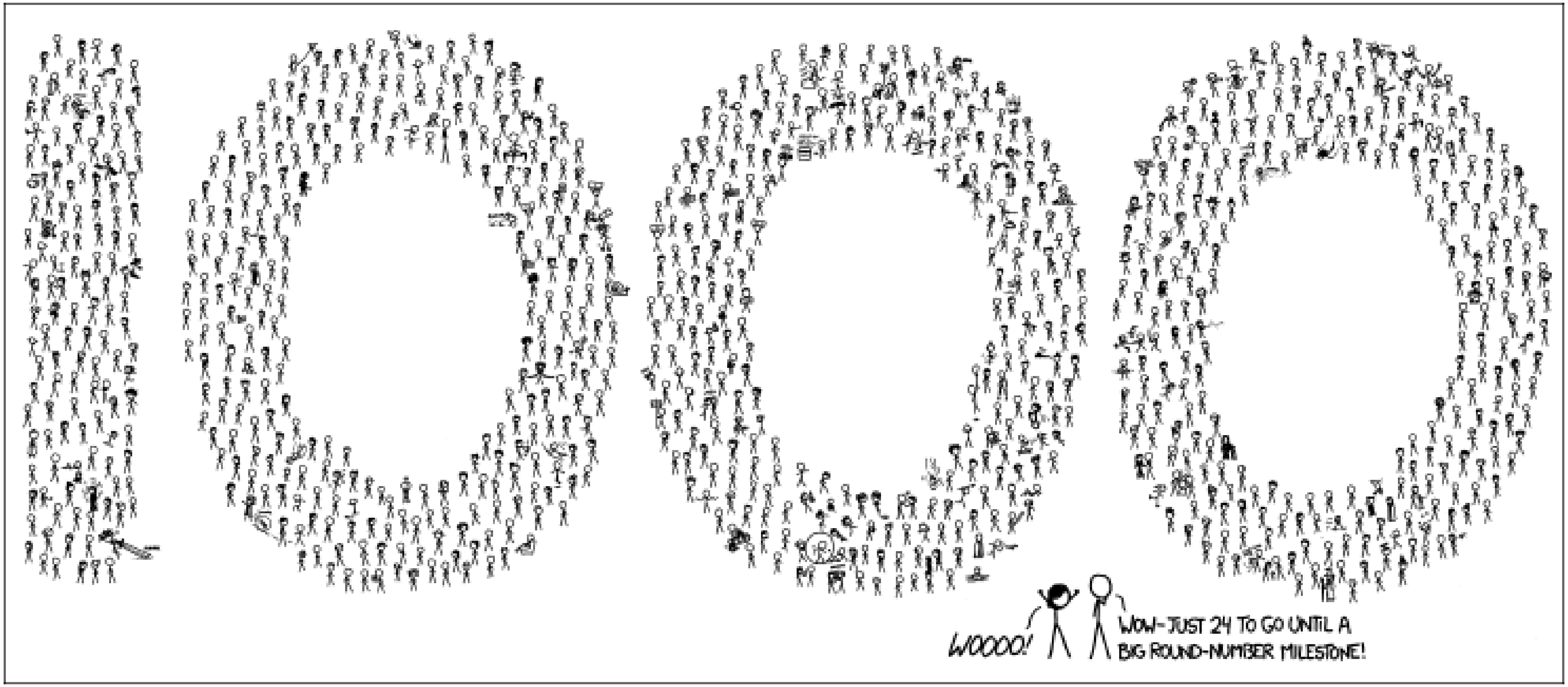
CSE 143

Lecture 28: Hashing



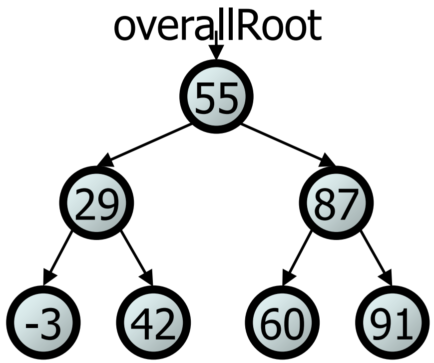
`SearchTree` as a set

- We implemented a class `SearchTree` to store a BST of `int`s:

- Our BST is essentially a set of integers.

  Operations we support:
  - `add`
  - `contains`
  - `remove`

  ...



- But there are other ways to implement a set...

How to implement a set?

- Elements of a `TreeSet` (`IntTree`) are in BST sorted order.
  - We need this in order to add or search in O(log *N*) time.

- But it doesn't really matter what order the elements appear in a set, so long as they can be added and searched quickly.

- Consider the task of storing a set in an array.
  - What would make a good ordering for the elements?

index	0	1	2	3	4	5	6	7	8	9
value	7	11	24	49	0	0	0	0	0	0

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	24	0	0	7	0	49

Hashing

- **hash**: To map a value to an integer index.
  - **hash table**: An array that stores elements via hashing.
- **hash function**: An algorithm that maps values to indexes.
  - one possible hash function for integers: **HF(I) → I % length**

```
set.add(11);       // 11 % 10 == 1
set.add(49);       // 49 % 10 == 9
set.add(24);       // 24 % 10 == 4
set.add(7);        //  7 % 10 == 7
```

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	24	0	0	7	0	49

Efficiency of hashing

```
public static int hashFunction(int i) {
    return Math.abs(i) % elementData.length;
}
```

- Add: set `elementData[HF(i)] = i;`
- Search: check if `elementData[HF(i)] == i`
- Remove: set `elementData[HF(i)] = 0;`

- What is the runtime of `add`, `contains`, and `remove`?
  - **O(1)!**

- Are there any problems with this approach?

Collisions

- **collision**: When hash function maps 2 values to same index.

```
set.add(11);
set.add(49);
set.add(24);
set.add(7);
set.add(54);   // collides with 24!
```

- **collision resolution**: An algorithm for fixing collisions.

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	54	0	0	7	0	49

Probing

- **probing**: Resolving a collision by moving to another index.
  - **linear probing**: Moves to the next index.

```
set.add(11);
set.add(49);
set.add(24);
set.add(7);
set.add(54);  // collides with 24; must probe
```

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	24	**54**	0	7	0	49

  - Is this a good approach?
    - variation: **quadratic probing** moves increasingly far away

Clustering

- **clustering**: Clumps of elements at neighboring indexes.
  - slows down the hash table lookup; you must loop through them.

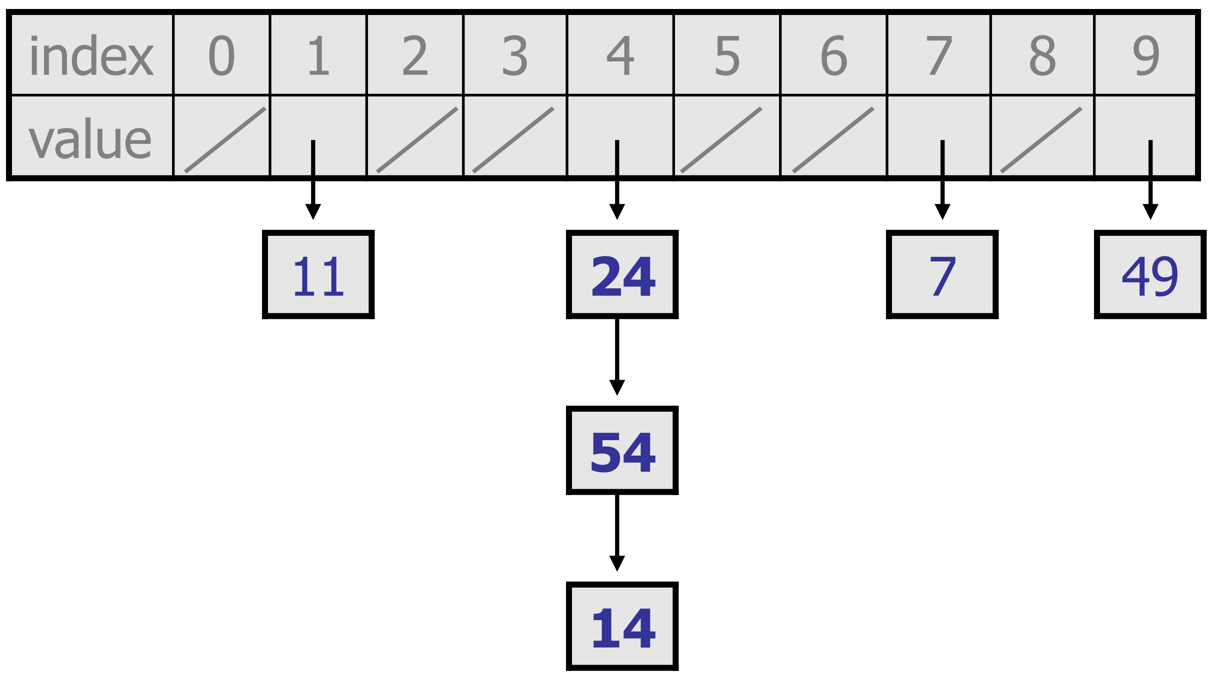
```
set.add(11);
set.add(49);
set.add(24);
set.add(7);
set.add(54);   // collides with 24
set.add(14);   // collides with 24, then 54
set.add(86);   // collides with 14, then 7
```

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	**24**	**54**	**14**	**7**	**86**	49

  - Now a lookup for 94 must look at 7 out of 10 total indexes.

Chaining

- **chaining**: Resolving collisions by storing a list at each index.
  - add/search/remove must traverse lists, but the lists are short
  - impossible to "run out" of indexes, unlike with probing



Hash set code

```
import java.util.*;   // for List, LinkedList

public class HashIntSet {
    private static final int CAPACITY = 137;
    private List<Integer>[] elements;

    // constructs new empty set
    public HashSet() {
        elements = (List<Integer>[]) (new List[CAPACITY]);
    }

    // adds the given value to this hash set
    public void add(int value) {
        int index = hashFunction(value);
        if (elements[index] == null) {
            elements[index] = new LinkedList<Integer>();
        }
        elements[index].add(value);
    }

    // hashing function to convert objects to indexes
    private int hashFunction(int value) {
        return Math.abs(value) % elements.length;
    }
    ...
```

Hash set code 2

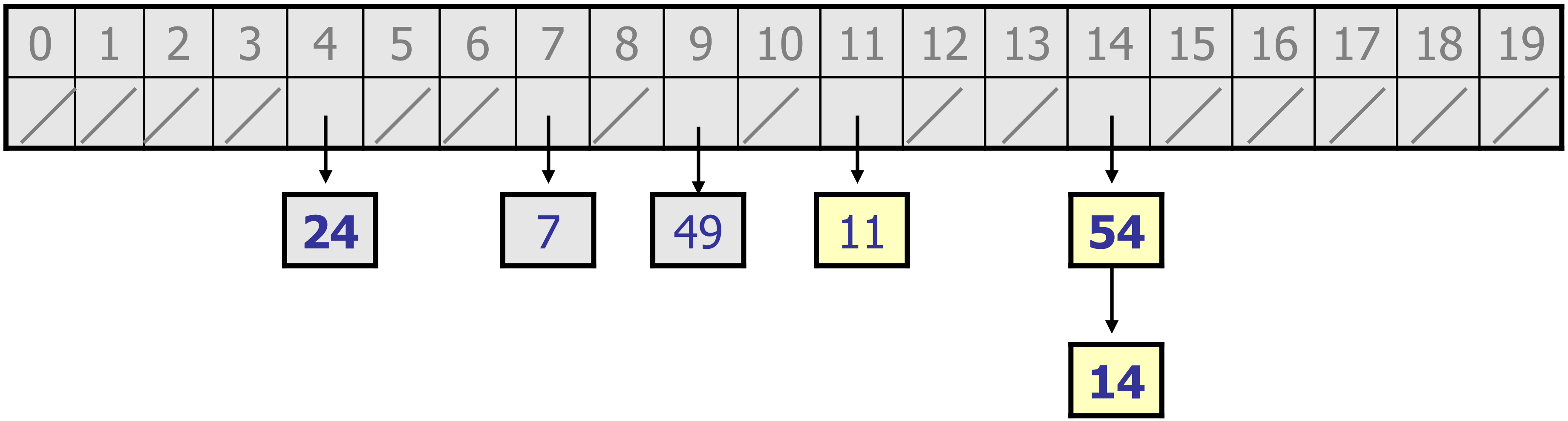
```
    ...
    // Returns true if this set contains the given value.
    public boolean contains(int value) {
        int index = hashFunction(value);
        return elements[index] != null &&
               elements[index].contains(value);
    }

    // Removes the given value from the set, if it exists.
    public void remove(int value) {
        int index = hashFunction(value);
        if (elements[index] != null) {
            elements[index].remove(value);
        }
    }
}
```

Rehashing

- **rehash**: Growing to a larger array when the table is too full.
  - Cannot simply copy the old array to a new one. (Why not?)

- **load factor**: ratio of (*# of elements*) / (*hash table length*)
  - many collections rehash when load factor ≅ .75
  - can use big prime numbers as hash table sizes to reduce collisions

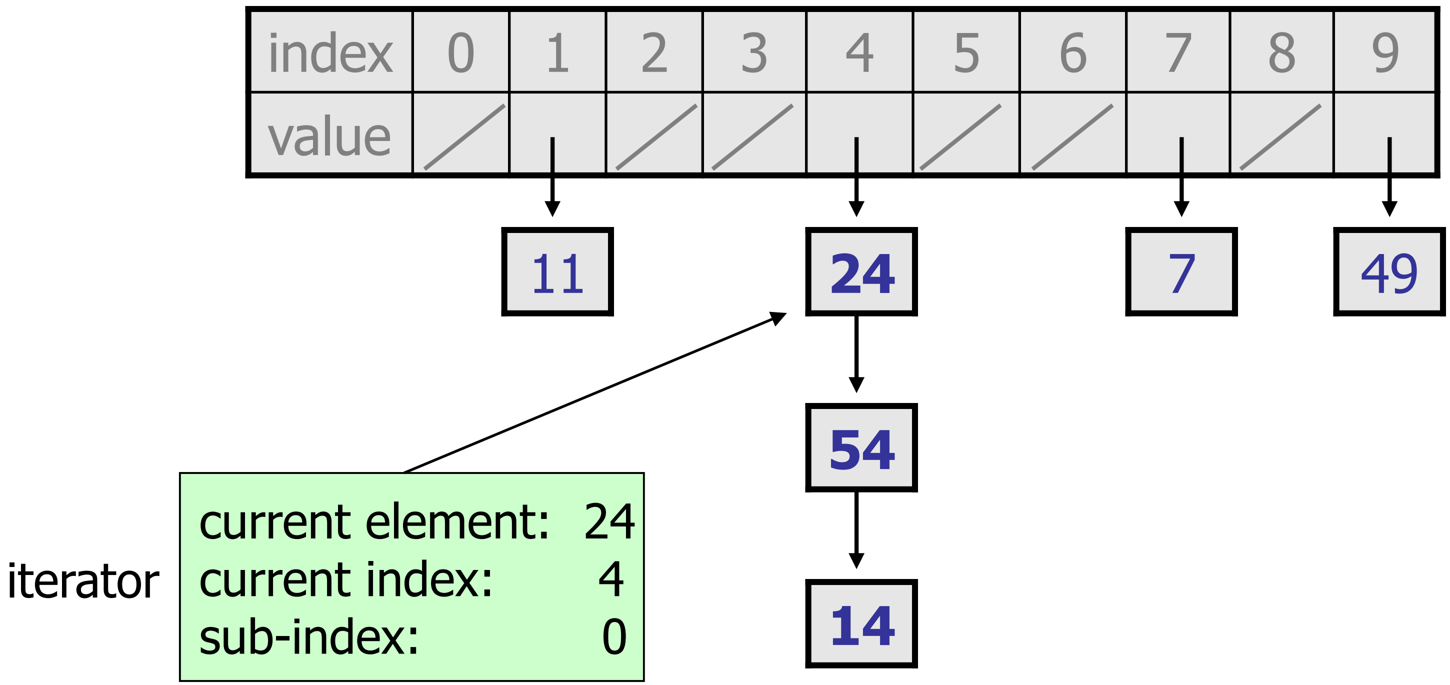


Rehashing code

```
...
// Grows hash array to twice its original size.
private void rehash() {
    List<Integer>[] oldElements = elements;
    elements = (List<Integer>[])
               new List[2 * elements.length];
    for (List<Integer> list : oldElements) {
        if (list != null) {
            for (int element : list) {
                add(element);
            }
        }
    }
}
```

Other questions

- How would we implement `toString` on a `HashSet`?
- How would we implement an `Iterator` over a `HashSet`?



Hashing objects

- It is easy to hash an integer I (use index *I % length*).
  - How can we hash other types of values (such as objects)?

- All Java objects contain the following method:

  ```
  public int hashCode()
  ```

  Returns an integer hash code for this object.

  - We can call `hashCode` on any object to find its preferred index.

- How is `hashCode` implemented?
  - Depends on the type of object and its state.
    - Example: a String's `hashCode` adds the ASCII values of its letters.
  - You can write your own `hashCode` methods in classes you write.
    - All classes come with a default version based on memory address.

Hash function for objects

```
public static int hashFunction(E e) {
    return Math.abs(e.hashCode()) % elements.length;
}
```

- Add: set `elements[HF(o)] = o;`
- Search: check if `elements[HF(o)].equals(o)`
- Remove: set `elements[HF(o)] = null;`

String's hashCode

- The `hashCode` function inside `String` objects looks like this:

```
public int hashCode() {
    int hash = 0;
    for (int i = 0; i < this.length(); i++) {
        hash = 31 * hash + this.charAt(i);
    }
    return hash;
}
```

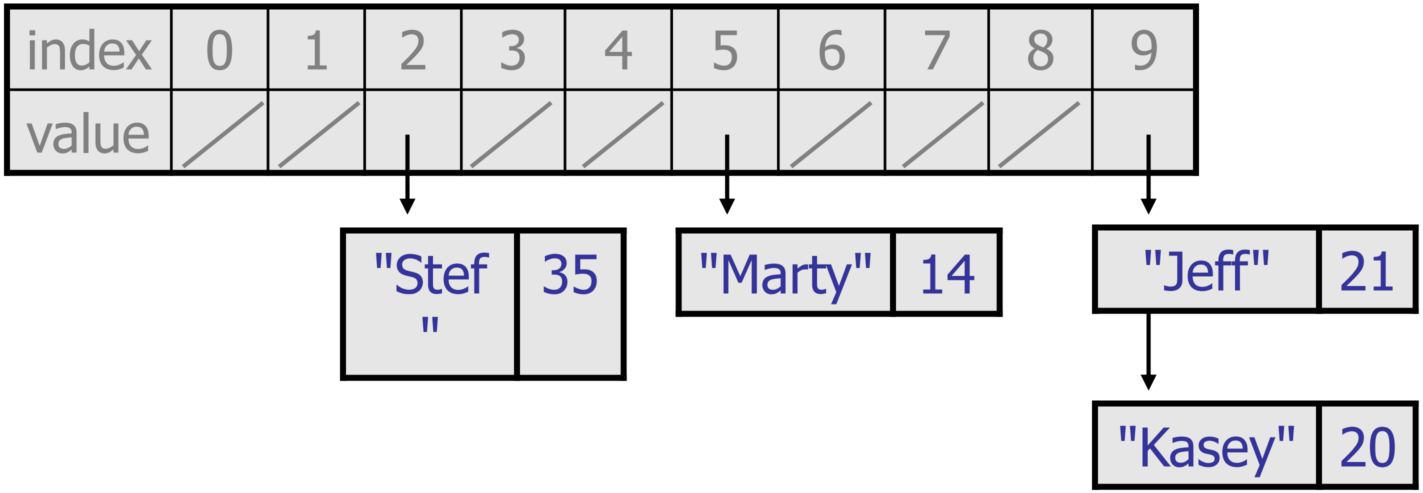
$$h(s) = \sum_{i=0}^{n-1} s[i] \cdot 31^{n-1-i}$$

- As with any general hashing function, collisions are possible.
  - Example: "Ea" and "FB" have the same hash value.
- Early versions of the Java examined only the first 16 characters. For some common data this led to poor hash table performance.

Implementing a hash map

- A hash map is just a set where the lists store key/value pairs:

```
//          key     value
map.put("Marty", 14);
map.put("Jeff",  21);
map.put("Kasey", 20);
map.put("Stef",  35);
```



  - Instead of a `List<Integer>`, write an inner `Entry` node class with `key` and `value` fields; the map stores a `List<Entry>`

Implementing a tree map

- Similar to difference between `HashMap` and `HashSet`:
  - Each node now will store both a key and a value
  - tree is BST ordered by keys
  - keys must be `Comparable`