

CSE 143

Lecture 18

Recursive Backtracking

reading: 12.5 or "Appendix R" on course web site

slides adapted from Marty Stepp and Hélène Martin

<http://www.cs.washington.edu/143/>

ideas and examples taken from Stanford University CS slides/lectures

Backtracking

- Useful to solve problems that require making decisions
 - Insufficient information to make a thoughtful choice
 - Each decision leads to new choices
 - Some sequence of choices will be a solution
- Backtracking involves trying out sequences of decisions until one that works is found
- Depth first search: we go deep down one path rather than broad
- Natural to implement recursively: call stack keeps track of decision points in right order (opposite from visited)

Exercise: Dice rolls

- Write a method `diceRoll` that accepts an integer parameter representing a number of 6-sided dice to roll, and output all possible combinations of values that could appear on the dice.

```
diceRoll(2);
```

[1, 1]	[3, 1]	[5, 1]
[1, 2]	[3, 2]	[5, 2]
[1, 3]	[3, 3]	[5, 3]
[1, 4]	[3, 4]	[5, 4]
[1, 5]	[3, 5]	[5, 5]
[1, 6]	[3, 6]	[5, 6]
[2, 1]	[4, 1]	[6, 1]
[2, 2]	[4, 2]	[6, 2]
[2, 3]	[4, 3]	[6, 3]
[2, 4]	[4, 4]	[6, 4]
[2, 5]	[4, 5]	[6, 5]
[2, 6]	[4, 6]	[6, 6]



```
diceRoll(3);
```

[1, 1, 1]
[1, 1, 2]
[1, 1, 3]
[1, 1, 4]
[1, 1, 5]
[1, 1, 6]
[1, 2, 1]
[1, 2, 2]
...
[6, 6, 4]
[6, 6, 5]
[6, 6, 6]

Examining the problem

- We want to generate all possible sequences of values.

for (each possible first die value):

for (each possible second die value):

for (each possible third die value):

...

print!



– This is called a **depth-first search**

– How can we completely explore such a large search space?

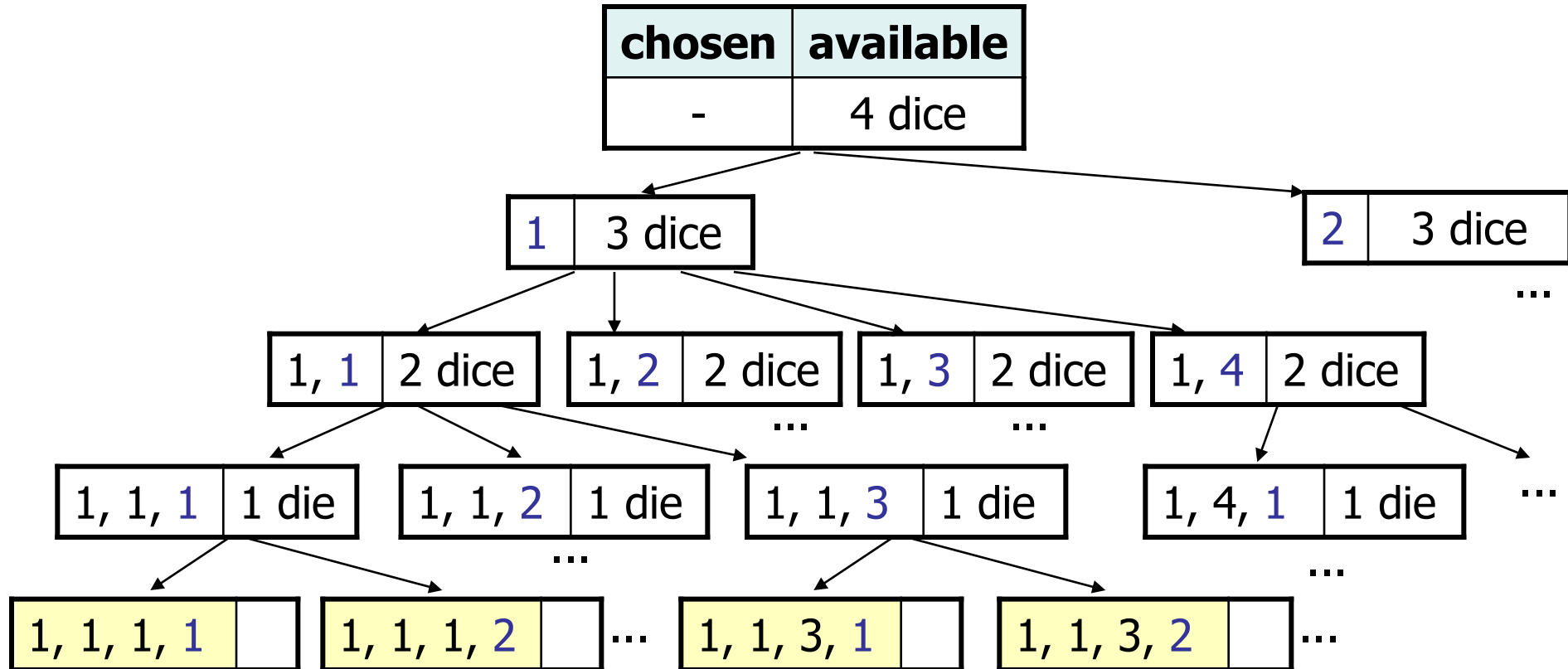
Backtracking algorithms

A general pseudo-code algorithm for backtracking problems:

Explore(**choices**):

- if there are no more **choices** to make: stop.
- else:
 - Make a single choice **C**.
 - Explore the remaining **choices**.
 - Un-make choice **C**, if necessary. (backtrack!)

A decision tree



Backtracking strategies

- When solving a backtracking problem, ask these questions:
 - What are the "choices" in this problem?
 - What is the "base case"? (How do I know when I'm out of choices?)
 - How do I "make" a choice?
 - Do I need to create additional variables to remember my choices?
 - Do I need to modify the values of existing variables?
 - How do I explore the rest of the choices?
 - Do I need to remove the made choice from the list of choices?
 - Once I'm done exploring, what should I do?
 - How do I "un-make" a choice?

Private helpers

- Often the method doesn't accept the parameters you want.
 - So write a **private helper** that accepts more parameters.
 - Extra params can represent current state, choices made, etc.

```
public int methodName (params) :  
    ...  
    return helper(params, moreParams) ;
```

```
private int helper(params, moreParams) :  
    ...  
    (use moreParams to help solve the problem)
```


Exercise solution

```
// Prints all possible outcomes of rolling the given
// number of six-sided dice in [#, #, #] format.
public static void diceRolls(int dice) {
    List<Integer> chosen = new ArrayList<Integer>();
    diceRolls(dice, chosen);
}

// private recursive helper to implement diceRolls logic
private static void diceRolls(int dice,
                                List<Integer> chosen) {
    if (dice == 0) {
        System.out.println(chosen);    // base case
    } else {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);              // choose
            diceRolls(dice - 1, chosen); // explore
            chosen.remove(chosen.size() - 1); // un-choose
        }
    }
}
```

Exercise: Dice roll sum

- Write a method `diceSum` similar to `diceRoll`, but it also accepts a desired sum and prints only combinations that add up to exactly that sum.

```
diceSum(2, 7);
```

```
[1, 6]  
[2, 5]  
[3, 4]  
[4, 3]  
[5, 2]  
[6, 1]
```

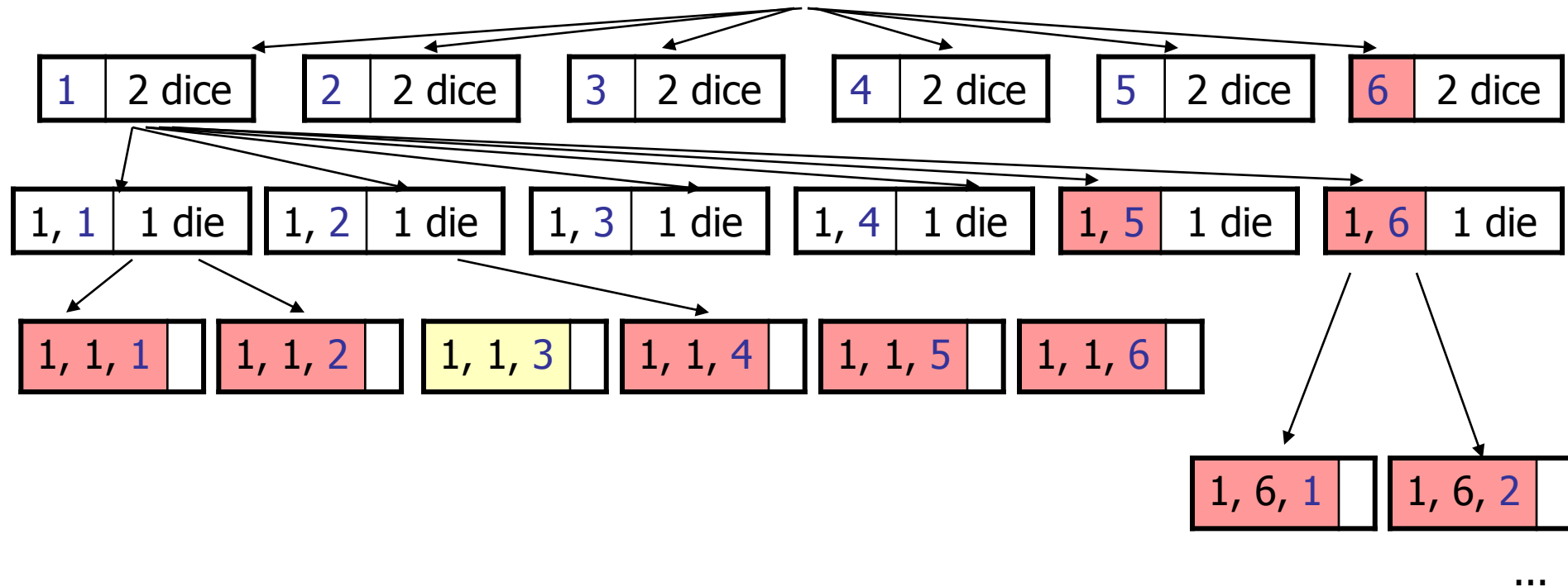


```
diceSum(3, 7);
```

```
[1, 1, 5]  
[1, 2, 4]  
[1, 3, 3]  
[1, 4, 2]  
[1, 5, 1]  
[2, 1, 4]  
[2, 2, 3]  
[2, 3, 2]  
[2, 4, 1]  
[3, 1, 3]  
[3, 2, 2]  
[3, 3, 1]  
[4, 1, 2]  
[4, 2, 1]  
[5, 1, 1]
```

New decision tree

chosen	available	desired sum
-	3 dice	5



Optimizations

- We need not visit every branch of the decision tree.
 - Some branches are clearly not going to lead to success.
 - We can preemptively stop, or **prune**, these branches.
- Inefficiencies in our dice sum algorithm:
 - Sometimes the current sum is already too high.
 - (Even rolling 1 for all remaining dice would exceed the desired sum.)
 - Sometimes the current sum is already too low.
 - (Even rolling 6 for all remaining dice would exceed the desired sum.)
 - When finished, the code must compute the sum every time.
 - $(1+1+1 = \dots, 1+1+2 = \dots, 1+1+3 = \dots, 1+1+4 = \dots, \dots)$

Exercise solution, improved

```
public static void diceSum(int dice, int desiredSum) {
    List<Integer> chosen = new ArrayList<Integer>();
    diceSum2(dice, desiredSum, chosen, 0);
}

private static void diceSum(int dice, int desiredSum,
    List<Integer> chosen, int sumSoFar) {
    if (dice == 0) {
        if (sumSoFar == desiredSum) {
            System.out.println(chosen);
        }
    } else if (sumSoFar <= desiredSum &&
        sumSoFar + 6 * dice >= desiredSum) {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);
            diceSum(dice - 1, desiredSum, chosen, sumSoFar + i);
            chosen.remove(chosen.size() - 1);
        }
    }
}
```

Exercise: Permutations

- Write a method `permute` that accepts a string as a parameter and outputs all possible rearrangements of the letters in that string. The arrangements may be output in any order.

– Example:

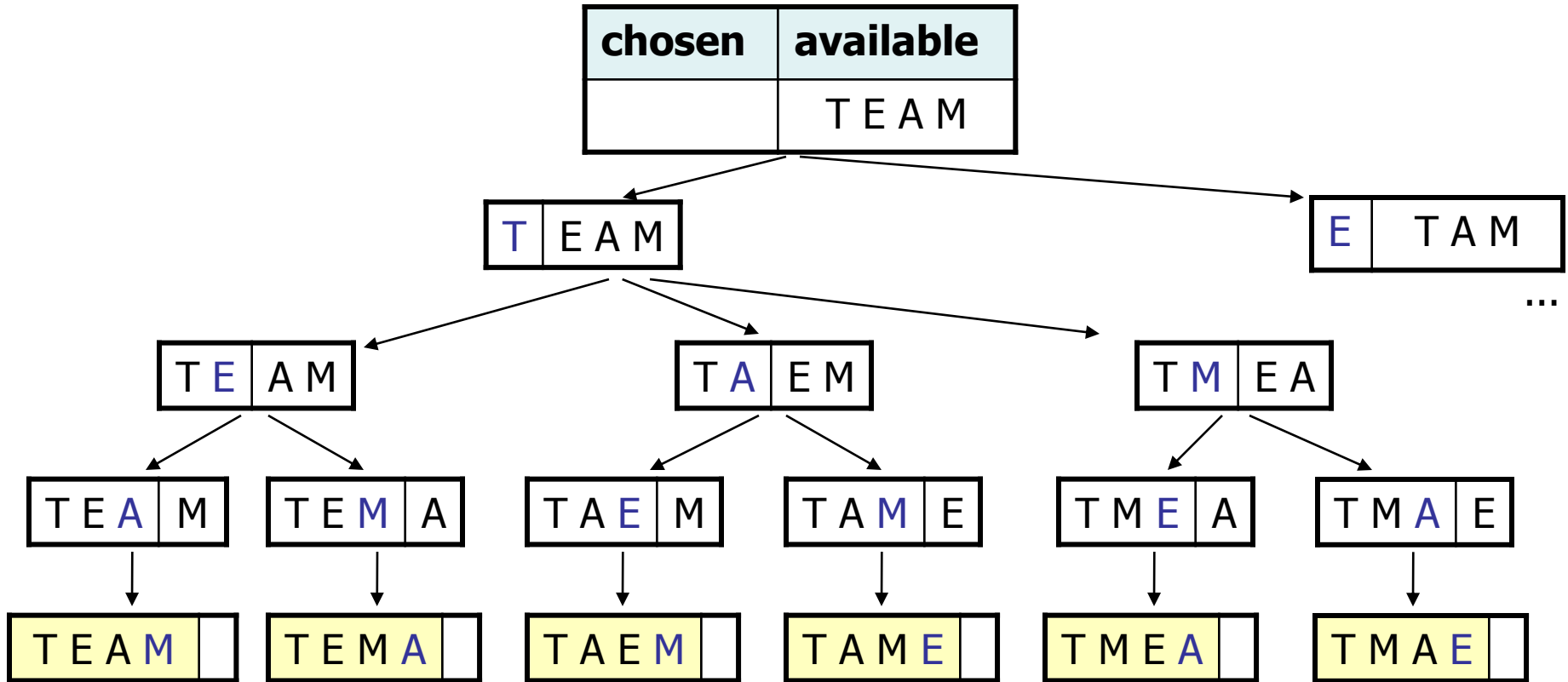
`permute("TEAM")`
outputs the following
sequence of lines:

TEAM	ATEM
TEMA	ATME
TAEM	AETM
TAME	AEMT
TMEA	AMTE
TMAE	AMET
ETAM	MTEA
ETMA	MTAE
EATM	META
EAMT	MEAT
EMTA	MATE
EMAT	MAET

Examining the problem

- We want to generate all possible sequences of letters.
 - for (each possible first letter):
 - for (each possible second letter):
 - for (each possible third letter):
 - ...
 - print!
- Each permutation is a set of choices or **decisions**:
 - Which character do I want to place first?
 - Which character do I want to place second?
 - ...
 - **solution space**: set of all possible sets of decisions to explore

Decision tree



Exercise solution

```
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            char c = s.charAt(i); // choose
            s = s.substring(0, i) + s.substring(i + 1);
            chosen += c;

            permute(s, chosen); // explore

            s = s.substring(0, i) + c + s.substring(i + 1);
            chosen = chosen.substring(0, chosen.length() - 1);
        } // un-choose
    }
}
```

Exercise solution 2

```
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1); // choose
            String rest = s.substring(0, i) + // remove
                s.substring(i + 1);
            permute(rest, chosen + ch); // explore
        }
    }
    // (don't need to "un-choose" because
    // we used temp variables)
}
```

Exercise: Combinations

- Write a method `combinations` that accepts a string s and an integer k as parameters and outputs all possible k -letter words that can be formed from unique letters in that string. The arrangements may be output in any order.

– Example:

```
combinations("GOOGLE", 3)
```

outputs the sequence of lines at right.

- To simplify the problem, you may assume that the string s contains at least k unique characters.

EGL	LEG
EGO	LEO
ELG	LGE
ELO	LGO
EOG	LOE
EOL	LOG
GEL	OEG
GEO	OEL
GLE	OGE
GLO	OGL
GOE	OLE
GOL	OLG

Initial attempt

```
public static void combinations(String s, int length) {
    combinations(s, "", length);
}

private static void combinations(String s, String chosen, int length) {
    if (length == 0) {
        System.out.println(chosen);    // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, length - 1);
            }
        }
    }
}
```

- Problem: Prints same string multiple times.

Exercise solution

```
public static void combinations(String s, int length) {
    Set<String> all = new TreeSet<String>();
    combinations(s, "", all, length);
    for (String comb : all) {
        System.out.println(comb);
    }
}

private static void combinations(String s, String chosen,
                                Set<String> all, int length) {
    if (length == 0) {
        all.add(chosen);           // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, all, length - 1);
            }
        }
    }
}
```