CSE 143
Lecture 12

Recursive Backtracking

reading: "Appendix R" on course web site

slides adapted from Marty Stepp and Hélène Martin

http://www.cs.washington.edu/143/

ideas and examples taken from Stanford University CS slides/lectures

Exercise: Dice rolls

- Write a method `diceRoll` that accepts an integer parameter representing a number of 6-sided dice to roll, and output all possible combinations of values that could appear on the dice.

```
diceRoll(2);

[1, 1]    [3, 1]    [5, 1]
[1, 2]    [3, 2]    [5, 2]
[1, 3]    [3, 3]    [5, 3]
[1, 4]    [3, 4]    [5, 4]
[1, 5]    [3, 5]    [5, 5]
[1, 6]    [3, 6]    [5, 6]
[2, 1]    [4, 1]    [6, 1]
[2, 2]    [4, 2]    [6, 2]
[2, 3]    [4, 3]    [6, 3]
[2, 4]    [4, 4]    [6, 4]
[2, 5]    [4, 5]    [6, 5]
[2, 6]    [4, 6]    [6, 6]
```

```
diceRoll(3);

[1, 1, 1]
[1, 1, 2]
[1, 1, 3]
[1, 1, 4]
[1, 1, 5]
[1, 1, 6]
[1, 2, 1]
[1, 2, 2]
    ...
[6, 6, 4]
[6, 6, 5]
[6, 6, 6]
```

Examining the problem

- We want to generate all possible sequences of values.

```
for (each possible first die value):
    for (each possible second die value):
        for (each possible third die value):
            ...
                print!
```

  - This is called a **depth-first search**

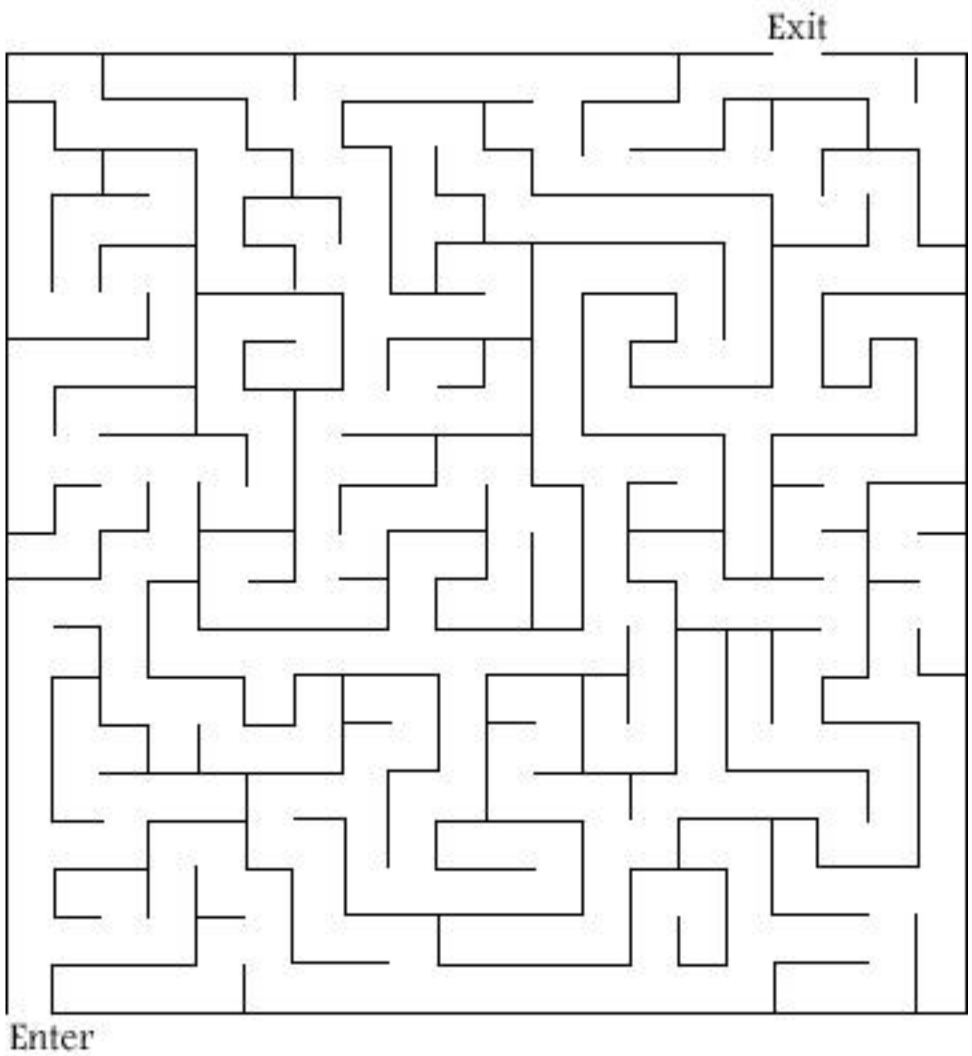
  - How can we completely explore such a large search space?

Backtracking

- **backtracking**: Finding solution(s) by trying partial solutions and then abandoning them if they are not suitable.
  - a "brute force" algorithmic technique (tries all paths)
  - often implemented recursively

  Applications:
  - producing all permutations of a set of values
  - parsing languages
  - games: anagrams, crosswords, word jumbles, 8 queens
  - combinatorics and logic programming

Backtracking



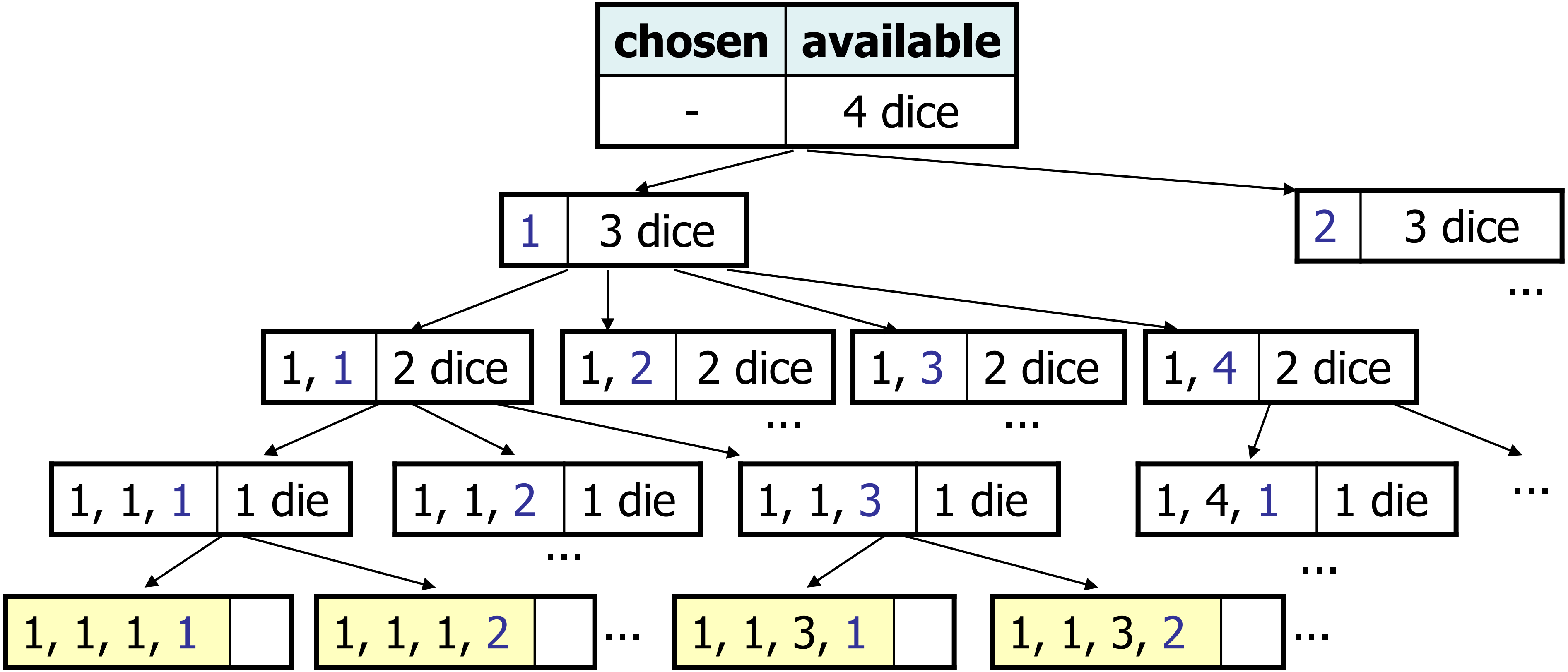
Backtracking algorithms

A general pseudo-code algorithm for backtracking problems:

Explore(**choices**):

- if there are no more **choices** to make: stop.
- else:
  - Make a single choice **C**.
  - Explore the remaining **choices**.
  - Un-make choice **C**, if necessary. (backtrack!)

A decision tree



Private helpers

- Often the method doesn't accept the parameters you want.
  - So write a **private helper** that accepts more parameters.
  - Extra params can represent current state, choices made, etc.

```
public int methodName(params):
    ...
    return helper(params, moreParams);


private int helper(params, moreParams):
    ...
    (use moreParams to help solve the problem)
```

Exercise solution

```
// Prints all possible outcomes of rolling the given
// number of six-sided dice in [#, #, #] format.
public static void diceRolls(int dice) {
    List<Integer> chosen = new ArrayList<Integer>();
    diceRolls(dice, chosen);
}

// private recursive helper to implement diceRolls logic
private static void diceRolls(int dice,
                              List<Integer> chosen) {
    if (dice == 0) {
        System.out.println(chosen);   // base case
    } else {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);                          // choose
            diceRolls(dice - 1, chosen);            // explore
            chosen.remove(chosen.size() - 1); // un-choose
        }
    }
}
```

Exercise: Dice roll sum

- Write a method `diceSum` similar to `diceRoll`, but it also accepts a desired sum and prints only combinations that add up to exactly that sum.

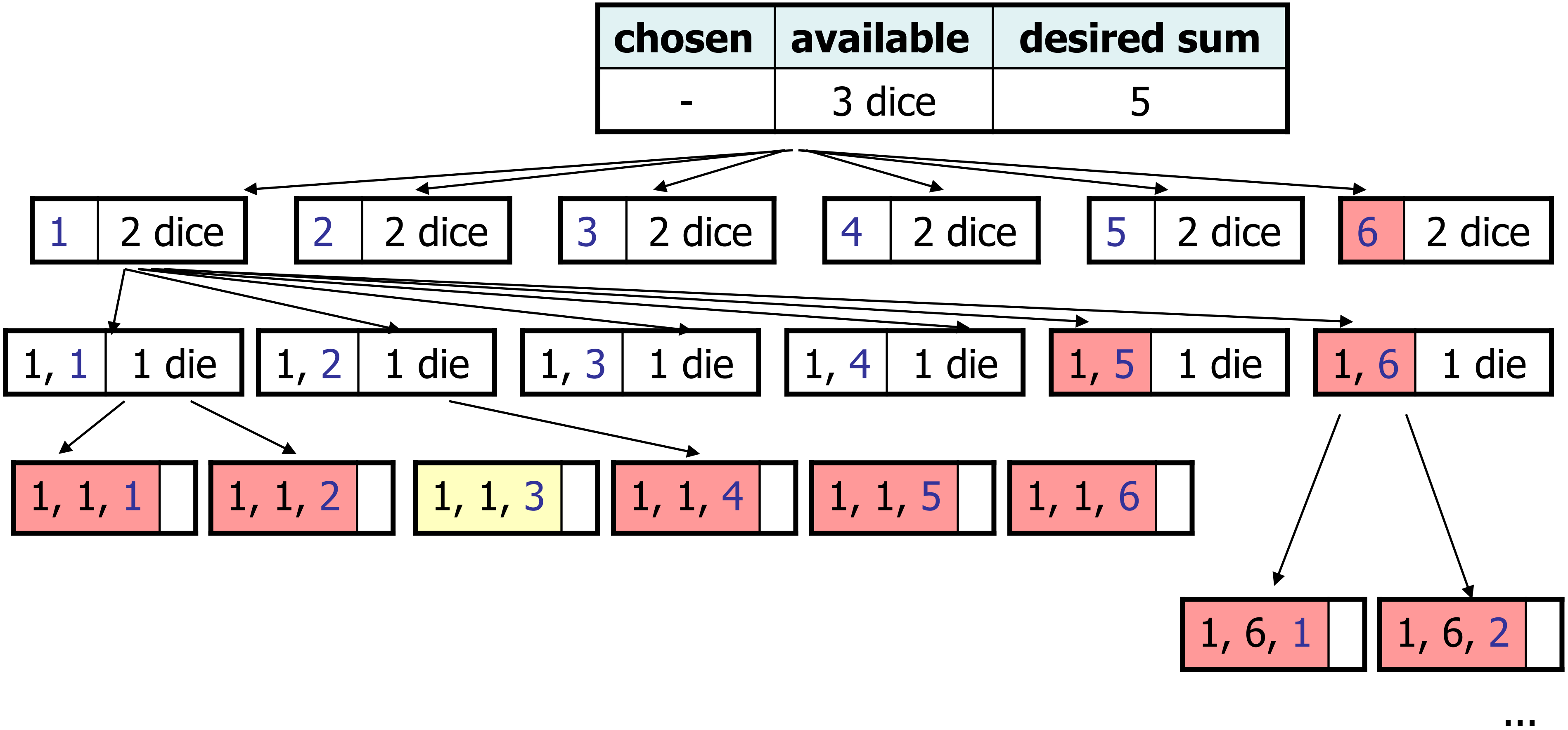
```
diceSum(2, 7);

    [1, 6]
    [2, 5]
    [3, 4]
    [4, 3]
    [5, 2]
    [6, 1]
```

```
diceSum(3, 7);

    [1, 1, 5]
    [1, 2, 4]
    [1, 3, 3]
    [1, 4, 2]
    [1, 5, 1]
    [2, 1, 4]
    [2, 2, 3]
    [2, 3, 2]
    [2, 4, 1]
    [3, 1, 3]
    [3, 2, 2]
    [3, 3, 1]
    [4, 1, 2]
    [4, 2, 1]
    [5, 1, 1]
```

New decision tree



Optimizations

- We need not visit every branch of the decision tree.
  - Some branches are clearly not going to lead to success.
  - We can preemptively stop, or **prune**, these branches.

- Inefficiencies in our dice sum algorithm:
  - Sometimes the current sum is already too high.
    - (Even rolling 1 for all remaining dice would exceed the desired sum.)
  - Sometimes the current sum is already too low.
    - (Even rolling 6 for all remaining dice would exceed the desired sum.)
  - When finished, the code must compute the sum every time.
    - (1+1+1 = ..., 1+1+2 = ..., 1+1+3 = ..., 1+1+4 = ..., ...)

Exercise solution, improved

```
public static void diceSum(int dice, int desiredSum) {
    List<Integer> chosen = new ArrayList<Integer>();
    diceSum(dice, desiredSum, chosen, 0);
}

private static void diceSum(int dice, int desiredSum,
                 List<Integer> chosen, int sumSoFar) {
    if (dice == 0) {
        if (sumSoFar == desiredSum) {
            System.out.println(chosen);
        }
    } else if (sumSoFar <= desiredSum &&
               sumSoFar + 6 * dice >= desiredSum) {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);
            diceSum(dice - 1, desiredSum, chosen, sumSoFar + i);
            chosen.remove(chosen.size() - 1);
        }
    }
}
```

Backtracking strategies

- When solving a backtracking problem, ask these questions:
  - What are the "choices" in this problem?
    - What is the "base case"? (How do I know when I'm out of choices?)
  - How do I "make" a choice?
    - Do I need to create additional variables to remember my choices?
    - Do I need to modify the values of existing variables?
  - How do I explore the rest of the choices?
    - Do I need to remove the made choice from the list of choices?
  - Once I'm done exploring, what should I do?
  - How do I "un-make" a choice?

Exercise: Permutations

- Write a method `permute` that accepts a string as a parameter and outputs all possible rearrangements of the letters in that string. The arrangements may be output in any order.
  - Example:
    `permute("TEAM")`
    outputs the following sequence of lines:

```
TEAM    ATEM
TEMA    ATME
TAEM    AETM
TAME    AEMT
TMEA    AMTE
TMAE    AMET
ETAM    MTEA
ETMA    MTAE
EATM    META
EAMT    MEAT
EMTA    MATE
EMAT    MAET
```

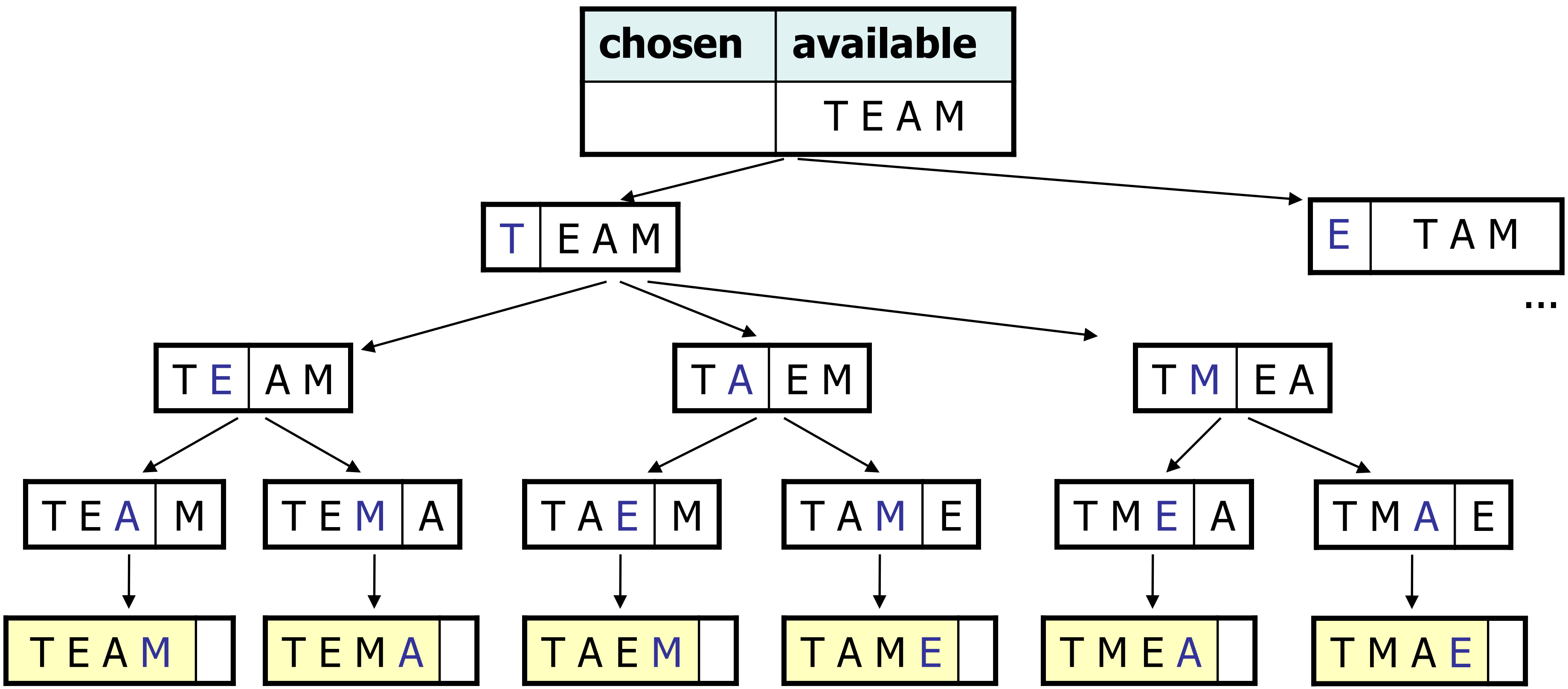
Examining the problem

- We want to generate all possible sequences of letters.

```
for (each possible first letter):
    for (each possible second letter):
        for (each possible third letter):
            ...
                print!
```

- Each permutation is a set of choices or **decisions**:
  - Which character do I want to place first?
  - Which character do I want to place second?
  - ...
  - **solution space**: set of all possible sets of decisions to explore

Decision tree



Exercise solution

```java
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            char c = s.charAt(i);                      // choose
            s = s.substring(0, i) + s.substring(i + 1);
            chosen += c;

            permute(s, chosen);                        // explore

            s = s.substring(0, i) + c + s.substring(i + 1);
            chosen = chosen.substring(0, chosen.length() - 1);
        }                                              // un-choose
    }
}
```

Exercise solution 2

```
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);  // choose

            String rest = s.substring(0, i) +   // remove
                          s.substring(i + 1);

            permute(rest, chosen + ch);         // explore
        }
    }       // (don't need to "un-choose" because
}           //  we used temp variables)
```

Exercise: Combinations

- Write a method `combinations` that accepts a string *s* and an integer *k* as parameters and outputs all possible *k*-letter words that can be formed from unique letters in that string. The arrangements may be output in any order.
  - Example:
    `combinations("GOOGLE", 3)`
    outputs the sequence of lines at right.
  - To simplify the problem, you may assume that the string *s* contains at least *k* unique characters.

EGL	LEG
EGO	LEO
ELG	LGE
ELO	LGO
EOG	LOE
EOL	LOG
GEL	OEG
GEO	OEL
GLE	OGE
GLO	OGL
GOE	OLE
GOL	OLG

Initial attempt

```
public static void combinations(String s, int length) {
    combinations(s, "", length);
}

private static void combinations(String s, String chosen, int length) {
    if (length == 0) {
        System.out.println(chosen);    // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, length - 1);
            }
        }
    }
}
```

– Problem: Prints same string multiple times.

Exercise solution

```
public static void combinations(String s, int length) {
    Set<String> all = new TreeSet<String>();
    combinations(s, "", all, length);
    for (String comb : all) {
        System.out.println(comb);
    }
}

private static void combinations(String s, String chosen,
                                 Set<String> all, int length) {
    if (length == 0) {
        all.add(chosen);          // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, all, length - 1);
            }
        }
    }
}
```