CSE 143
Lecture 15

Binary Search; `Comparable`

reading: 13.2; 10.2

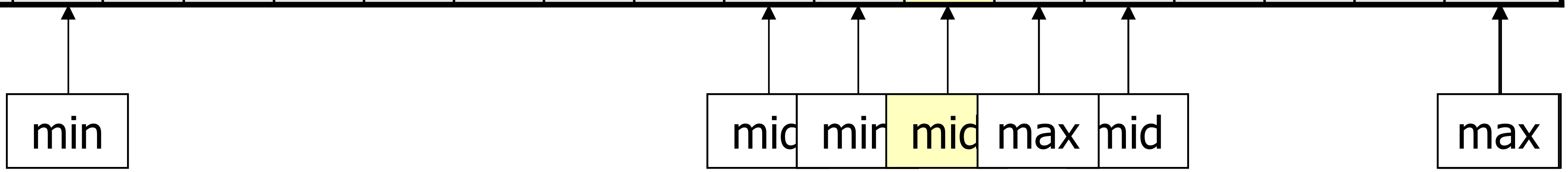
slides created by Marty Stepp
http://www.cs.washington.edu/143/

Binary search (13.1)

- **binary search**: Locates a target value in a sorted array/list by successively eliminating half of the array from consideration.
  - O(log N) runtime for an array of size N
  - can be implemented iteratively (with a loop) or recursively

  - Example: Searching the array below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



Binary search code

```
// Returns the index of an occurrence of target in a,
// or a negative number if the target is not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(int[] a, int target) {
    int min = 0;
    int max = a.length - 1;

    while (min <= max) {
        int mid = (min + max) / 2;
        if (a[mid] < target) {
            min = mid + 1;
        } else if (a[mid] > target) {
            max = mid - 1;
        } else {
            return mid;   // target found
        }
    }

    return -(min + 1);    // target not found
}
```

Recursive binary search (13.3)

- Write a method `binarySearch` that accepts a sorted array of integers and a target integer value and returns the index of an occurrence of that target value in the array.
  - If the target value is not found, return a negative number.
  - Write the method recursively.

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

```
int index  = binarySearch(data, 42);  // 10
int index2 = binarySearch(data, 66);  // -1 or -14
```

Exercise solution

```java
// Returns the index of an occurrence of the given value in
// the given array, or a negative number if not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(int[] a, int target) {
    return binarySearch(a, target, 0, a.length - 1);
}

// Recursive helper to implement search behavior.
private static int binarySearch(int[] a, int target,
                                int min, int max) {
    if (min > max) {
        return -1;        // target not found
    } else {
        int mid = (min + max) / 2;
        if (a[mid] < target) {         // too small; go right
            return binarySearch(a, target, mid + 1, max);
        } else if (a[mid] > target) {  // too large; go left
            return binarySearch(a, target, mid + 1, max);
        } else {
            return mid;   // target found; a[mid] == target
        }
    }
}
```

Binary search and objects

- Suppose we want to modify the `binarySearch` method to search an array of Strings.
  - Operators like < and > do not work with `String` objects.
  - But we do think of strings as having an alphabetical ordering.

- **natural ordering**: Rules governing the relative placement of all values of a given type.

- **comparison function**: Code that, when given two values *A* and *B* of a given type, decides their relative ordering:
  - A < B, A == B, A > B

The `compareTo` method (10.2)

- The standard way for a Java class to define a comparison function for its objects is to define a `compareTo` method.
  - Example: in the `String` class, there is a method:

    `public int compareTo(String other)`

- A call of **a**`.compareTo(`**b**`)` will return:

a value <	0	if **a** comes "before" **b** in the ordering,
a value >	0	if **a** comes "after" **b** in the ordering,
or	0	if **a** and **b** are considered "equal" in the ordering.

Using compareTo

- `compareTo` can be used as a test in an `if` statement.

```
String a = "alice";
String b = "bob";
if (a.compareTo(b) < 0) {  // true
    ...
}
```

Primitives	Objects
`if (a < b) { ...`	`if (a.compareTo(b) < 0) { ...`
`if (a <= b) { ...`	`if (a.compareTo(b) <= 0) { ...`
`if (a == b) { ...`	`if (a.compareTo(b) == 0) { ...`
`if (a != b) { ...`	`if (a.compareTo(b) != 0) { ...`
`if (a >= b) { ...`	`if (a.compareTo(b) >= 0) { ...`
`if (a > b) { ...`	`if (a.compareTo(b) > 0) { ...`

Binary search w/ strings

```
// Returns the index of an occurrence of target in a,
// or a negative number if the target is not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(String[] a, int target) {
    int min = 0;
    int max = a.length - 1;

    while (min <= max) {
        int mid = (min + max) / 2;
        if (a[mid].compareTo(target) < 0) {
            min = mid + 1;
        } else if (a[mid].compareTo(target) > 0) {
            max = mid - 1;
        } else {
            return mid;   // target found
        }
    }

    return -(min + 1);    // target not found
}
```

compareTo and collections

- You can use an array or list of Strings with Java's included binary search method because it calls `compareTo` internally.

```
String[] a = {"al", "bob", "cari", "dan", "mike"};
int index = Arrays.binarySearch(a, "dan");  // 3


Set<String> set = new TreeSet<String>();
for (String s : a) {
    set.add(s);
}
System.out.println(s);
// [al, bob, cari, dan, mike]
```

Ordering our own types

- We cannot use `binarySearch` or make a `TreeSet/Map` of any arbitrary type, because Java doesn't know how to order the elements.
  - The program compiles but crashes when we run it.

```
Set<HtmlTag> tags = new TreeSet<HtmlTag>();
tags.add(new HtmlTag("body", true));
tags.add(new HtmlTag("b", false));
...

Exception in thread "main" java.lang.ClassCastException
        at java.util.TreeSet.add(TreeSet.java:238)
```

Comparable (10.2)

```
public interface Comparable<E> {
    public int compareTo(E other);
}
```

- A class can implement the `Comparable` interface to define a natural ordering function for its objects.

- A call to your `compareTo` method should return:

  a value < 0   if the `other` object comes "before" `this` one,
  a value > 0   if the `other` object comes "after" `this` one,
  or   0   if the `other` object is considered "equal" to `this`.

- If you want multiple orderings, use a `Comparator` instead (see Ch. 13.1)

Comparable template

```
public class name implements Comparable<name> {

    ...

    public int compareTo(name other) {
        ...
    }
}
```

Comparable example

```
public class Point implements Comparable<Point> {
    private int x;
    private int y;
    ...

    // sort by x and break ties by y
    public int compareTo(Point other) {
        if (x < other.x) {
            return -1;
        } else if (x > other.x) {
            return 1;
        } else if (y < other.y) {
            return -1;   // same x, smaller y
        } else if (y > other.y) {
            return 1;    // same x, larger y
        } else {
            return 0;    // same x and same y
        }
    }
}
```

`compareTo` tricks

- *subtraction trick* - Subtracting related numeric values produces the right result for what you want `compareTo` to return:

```
// sort by x and break ties by y
public int compareTo(Point other) {
    if (x != other.x) {
        return x - other.x;   // different x
    } else {
        return y - other.y;   // same x; compare y
    }
}
```

  - The idea:
    - if `x > other.x`, then `x - other.x > 0`
    - if `x < other.x`, then `x - other.x < 0`
    - if `x == other.x`, then `x - other.x == 0`

  - NOTE: This trick doesn't work for `doubles` (but see `Math.signum`)

`compareTo` tricks 2

- *delegation trick* - If your object's fields are comparable (such as strings), use their `compareTo` results to help you:

```
// sort by employee name, e.g. "Jim" < "Susan"
public int compareTo(Employee other) {
    return name.compareTo(other.getName());
}
```

- *`toString` trick* - If your object's `toString` representation is related to the ordering, use that to help you:

```
// sort by date, e.g. "09/19" > "04/01"
public int compareTo(Date other) {
    return toString().compareTo(other.toString());
}
```

Exercises

- Make the `Person` class from Homework 3 comparable.
  - Compare people alphabetically by name, case-insensitively.
- Make the `HtmlTag` class from Homework 2 comparable.
  - Compare tags by their elements, alphabetically by name.
  - If two tags have the same element, opening tags should come before closing tags.