

## CSE 143 Java

### Searching and Recursion

*Reading: Ch. 13 & Secs. 17.1-17.3*

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## Overview

### • Topics

- Maintaining an ordered list
- Sequential and binary search
- Recursion
- Divide and conquer strategy

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## Problem: A Word Dictionary

- Suppose we want to maintain a real dictionary. Data is a list of <word, definition> pairs -- a "Map" structure

<"aardvark", "an animal that starts with an A and ends with a K">

<"apple", "a leading product of Washington state">

<"banana", "a fruit imported from somewhere else">

etc.

- We want to be able to do the following operations efficiently

- Look up a definition given a word (key)
- Retrieve sequences of definitions in alphabetical order

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## Representation

- Need to pick a data structure
- Analyze possibilities based on cost of operations

search                  access next in order

- unordered list

Array?

Linked List?

- ?

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## Sequential (Linear) Search

- If we don't know anything about the list, we can use a *linear search* to locate a word

```
// return location of word in words, or -1 if found
int find(String word) {
    int k = 0;
    while (k < size && !word.equals(words[k])) {
        k++;
    }
    if (k < size) { return k; } else { return -1; } // lousy indenting to fit on slide
} // don't do this at home
```

- Time for list of size n:
  - Can we do better?

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## Can we do better?

- Yes if the list is in alphabetical order
- To simplify the explanations for the present: we'll treat the list as an array of strings

```
0 aardvark // instance variable of the Ordered List class
1 apple   String[] words; // list is stored in words[0..size-1]
2 banana  int size; // # of words
3 cherry
4 kumquat
5 orange
6 pear
7 rutabaga
```

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## Binary Search

- Key idea: to search a section of the array
  - Examine middle element
  - Search either left or right half depending on whether desired word precedes or follows middle word alphabetically
- The list being sorted is a *precondition* of binary search.
  - The algorithm is not guaranteed to give the correct answer if the precondition is violated

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## Binary Search

```
// Return location of word in words, or -1 if not found
int find(String word) {
    return bSearch(word, 0, size-1);
}
// Return location of word in words[lo..hi] or -1 if not found
int bSearch(String word, int lo, int hi) {
    // return -1 if interval lo..hi is empty
    if (lo > hi) { return -1; }
    // search words[lo..hi]
    int mid = (lo + hi) / 2;
    int comp = word.compareTo(words[mid]);
    if (comp == 0) { return mid; }
    else if (comp < 0) { return _____; }
    else /* comp > 0 */ { return _____; }
}
```

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## "The Word Must Be Where?" Three Cases

```
int comp = word.compareTo(words[mid]);
if (comp == 0) {
    //the word must be where? _____
    return _____;
}
else if (comp < 0) {
    //the word must be where? _____
    return _____;
}
else { //comp > 0
    //the word must be where? _____
    return _____;
}
```

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## "Where?" Answered

```
int comp = word.compareTo(words[mid]);
if (comp == 0) {
    //the word must be where? at position "mid"
    return _____;
}
else if (comp < 0) {
    //the word must be where? in the lower half of the array
    return _____;
}
else { //comp > 0
    //the word must be where? in the upper half of the array
    return _____;
}
```

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## Return Values: Three Cases

```
int comp = word.compareTo(words[mid]);
if (comp == 0) {
    //the word must be where? at position "mid"
    return mid;
}
else if (comp < 0) {
    //the word must be where? in the lower half of the array
    return /"the result of searching the lower half of the array"/
}
else { //comp > 0
    //the word must be where? in the upper half of the array
    return /"the result of searching the upper half of the array"/
}
```

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## What is "The Lower Half"?

```
... else if (comp < 0) {
    //the word must be where? in the lower half of the array
    return /"the result of searching the lower half of the array"/
}
...
```

Remember the method header was:  
// Return location of word in words[lo..hi] or -1 if not found  
int bSearch(String word, int lo, int hi) {

So the lower half starts at \_\_\_\_\_ and ends at \_\_\_\_\_  
return /"the result of searching the lower half of the array"/ becomes  
return /"the result of searching the array from \_\_\_\_ to \_\_\_\_"/

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## Comments Complete, Code Incomplete

```
int comp = word.compareTo(words[mid]);
if (comp == 0) {
    //the word must be where? at position "mid"
    return mid;
}
else if (comp < 0) {
    //the word must be where? in the lower half of the array
    return //the result of searching from lo to mid-1'/
}
else { //comp > 0
    //the word must be where? in the upper half of the array
    return //the result of searching from mid+1 to hi'/
}
```

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## Last Piece of the Puzzle

```
...
return //the result of searching from lo to mid-1'/
};
}
```

How can we get the "result of searching from lo to mid-1"?

We have a method called bSearch that can search an array within a range of indexes.

```
// Return location of word in words[x.y] or -1 if not found
int bSearch(String word, int x, int y)
```

Let x be lo, let y be mid-1  
bSearch(String word, int lo, int mid-1)

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## Recursion

- A method (function) that calls itself is *recursive*
- Nothing really new here
- Method call review:
  - Evaluate argument expressions
  - Allocate space for parameters and local variables of function being called
  - Initialize parameters with argument values
  - Then execute the function body
- What if the function being called is the same one that is doing the calling?
  - Answer: no difference at all!

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## Wrong Way to Think About It



- **Not** like a reflection in a mirror
- More like a clone that you send off to do an errand and bring you the results
- Definitely not like a snake swallowing itself...

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**Recursion**

The wrong way to think about it...

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### Right Way to Think About It

bSearch

```

graph TD
    A["...  
bSearch(array, lo, mid-1)  
..."] -- bSearch --> B["...  
bSearch(array, lo, mid-1)  
..."]
    B -- "Send in the clones..." --> C["..."]
  
```

"Send in the clones..."

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### Trace

- Trace execution of find("orange")
  - 0 aardvark
  - 1 apple
  - 2 banana
  - 3 cherry
  - 4 kumquat
  - 5 orange
  - 6 pear
  - 7 rutabaga

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### Trace

- Trace execution of find("kiwi")
  - 0 aardvark
  - 1 apple
  - 2 banana
  - 3 cherry
  - 4 kumquat
  - 5 orange
  - 6 pear
  - 7 rutabaga

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## Performance of Binary Search

- **Analysis**
  - Time (number of steps) per each recursive call:
  - Number of recursive calls:
  - Total time:
- A picture helps

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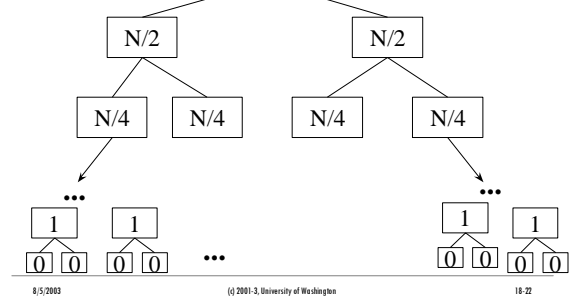
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## Binary Search Sizes

All paths from the size  $N$  case to a size 0 case are the same length:  $1 + \log_2 N$

Any given run of binary search will follow only one path from the root to a leaf



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## Linear Search vs. Binary Search

- **Compare to linear search**
  - Time to search 10, 100, 1000, 1,000,000 words
    - linear
    - binary
  - What is incremental cost if size of list is doubled?
- **Why is Binary search faster?**
  - The data structure is the same
  - The precondition on the data structure is different: stronger
  - Recursion itself is *not* an explanation
    - One could code linear search using recursion, or binary search without

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## More About Recursion

A recursive function needs three things to work properly

1. One or more **base cases** that are not recursive
  - if ( $lo > hi$ ) { return -1; }
  - if ( $comp == 0$ ) { return mid; }
2. One or more **recursive cases** that handle a situation by calling the method again
  - else if ( $comp > 0$ ) { return bsearch(word, mid+1, hi); }
3. The recursive cases must lead to "smaller" instances of the problem
  - "Smaller" means: closer to a base case
  - Without "smaller", what might happen?

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## "Divide and Conquer"

- Takes a cue from military folklore
- Three-step strategy
  1. Divide the problem into smaller subproblems
  2. Solve the subproblems separately
    - Using recursion, of course!
  3. Combine the subproblem solutions into the overall solution

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## Recursion vs. Iteration: The SmackDown

- |  |   |
|--|---|
| <ul style="list-style-type: none"><li>• Recursion can completely replace iteration</li><li>• Some rewriting of the iterative algorithm is necessary<ul style="list-style-type: none"><li>• usually minor</li></ul></li><li>• Some languages have recursion only</li><li>• Recursion is often more elegant but has some extra overhead</li><li>• Recursion is a natural for certain algorithms and data structures (where branching is required)<ul style="list-style-type: none"><li>• very effective in "divide and conquer" situations</li></ul></li></ul> | <ul style="list-style-type: none"><li>• Iteration can completely replace recursion</li><li>• Some rewriting of the recursive algorithm is necessary<ul style="list-style-type: none"><li>• often major</li></ul></li><li>• A few (mostly older languages) have iteration only</li><li>• Iteration is not always elegant but is usually efficient</li><li>• Iteration is natural for linear (non-branching) algorithms and data structures</li></ul> |
|--|---|

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## Recursion and Elegance

- Problem: reverse a linked list
- Constraints: no *last* pointer, no *numElems* count, no backward pointers, no additional data structures
- Footnote: this is known as a Microsoft job interview question (really!)
- Non-recursive solution: You try to find it

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## Result of Trying To Reverse a Linked List Iteratively

- Problem: reverse a linked list
- Constraints: no *last* pointer, no *numElems* count, no backward pointers, no additional data structures
- Non-recursive solution:
  - try it and weep



- Better hope this wasn't a question on *your* Microsoft job interview!

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## Recursive Solution: Simple, Elegant

- Problem: reverse a linked list
- Constraints: no *last* pointer, no *numElems* count, no backward pointers, no additional data structures

```
newList = reverse(oldList.first);
```

```
...
```

```
List reverse(Link firstLink) {  
    if (firstLink == null) { return new SimpleList(); }  
    return reverse(firstLink.next).add(firstLink.data);  
}
```

- Better hope this *is* a question on your Microsoft job interview!
- PS: Did we cheat??

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## Summary

### • Recursion

- Methods that call themselves
- Need base case(s) and recursive case(s)
- Recursive cases need to progress toward a base case
- Often a very clean way to formulate a problem (let the function call mechanism handle bookkeeping behind the scenes)

### • Divide and Conquer

- Algorithm design strategy that exploits recursion
- Divide original problem into subproblems
- Solve each subproblem recursively
- Can sometimes yield dramatic performance improvements

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