CSE 143 Java

Program Efficiency &
Introduction to Complexity Theory

Reading: Ch. 21 (go light on the math)

GREAT IDEAS IN COMPUTER SCIENCE

ANALYSIS OF ALGORITHMIC COMPLEXITY

Overview

- Measuring time and space used by algorithms
- Machine-independent measurements
- Costs of operations
- Asymptotic complexity – O( ) notation and complexity classes
- Comparing algorithms
- Performance tuning

Comparing Algorithms

- Example: We've seen two different list implementations
  - Dynamic expanding array
  - Linked list
- Which is "better"?
- How do we measure?
  - Stopwatch? Why or why not?

Program Resources & Efficiency

- Running a program has "costs"
- A program uses computer "resources"
  - Execution time
  - Execution space
  - Network bandwidth
  - others
- Goal: Find way to measure "resource" usage in a way that is independent of particular machines or implementations
- We will focus on execution time
  - Techniques/vocabulary apply to other resource measures

Example

- What is the running time of the following method?

```
// Return the sum of the elements in array.
double sum(double[ ] data) {
    double ans = 0.0;
    for (int k = 0; k < data.length; k++) {
        ans = ans + data[k];
    }
    return ans;
}
```

- How do we analyze this?
- What does the question even mean?

```
double sum(double[ ] data) {
    double ans = 0.0;
    for (int k = 0; k < data.length; k++) {
        ans = ans + data[k];
    }
    return ans;
}
```


Analysis of Execution Time

1. First: describe the *size* of the problem in terms of one or more parameters
   - For *sum*, size of array makes sense
   - Often size of data structure, but can be magnitude of some numeric parameter, etc.
2. Then, count the number of *steps* needed *as a function of the problem size*

- Need to define what a "step" is
  - First approximation: one simple statement
  - More complex statements will be multiple steps

What is a "Step?"

- First approximation:
  - one step = one Java statement
- Better: *execution* of one Java statement
  - Take into account statements which are repeated (in loops)
  - Take into account statements which are skipped (in conditionals)
  - Take into account work that is performed in advance (by compiler instead of at run-time)
- Different statement categories may enclose different numbers of steps
  - Consider each type separately: constant-time operations, zero-time ops, conditionals, loops, methods calls, etc.

Cost of operations: **Constant Time Ops**

- Constant-time operations: each take one abstract time "step"
  - Simple variable declaration/initialization (double sum = 0.0;)
  - Assignment of numeric or reference values (var = value;)
  - Arithmetic operation (+, -, *, /, %)
  - Array subscripting (a[index])
  - Simple conditional tests (x < y, p != null)
  - Operator new itself (not including constructor cost)

    Note: new takes significantly longer than simple arithmetic or assignment, but its cost is independent of the problem we're trying to analyze
- Note: watch out for things like method calls or constructor invocations that look simple, but can be expensive

Cost of operations: **Zero-time Ops**

- Can sometimes perform operations at compile time
  - Nothing left to do at runtime
- Variable declarations without initialization

  double[ ] overdrafts;
- Variable declarations with compile-time constant initializers

  static final int maxButtons = 3;
- Some casts (but not those that need a runtime check)

  int code = (int) '?';

Cost of operations: **Sequences of Statements**

- Cost of

  S1; S2; ...; Sn

  is sum of the costs of S1 + S2 + … + Sn

Cost of operations: **Conditional Statement**

- **We're generally trying to figure out how long it *might* take to execute a statement (*worst case*), so the cost of**

```
if (condition) {
    S1;
} else {
    S2;
}
```

is normally the max cost of S1 or S2 (plus cost of the condition)

- **Other possibilities (less common)**
  - ***Best case* – use the min cost of S1 or S2**
  - ***Expected (average) case* – probabilistic analysis needed**

Cost of operations: **Analyzing Loops**

- **Basic analysis**
  1. **Calculate cost of each iteration**
  2. **Calculate number of iterations**
  3. **Total cost is the product of these**

     Caution -- need to add up the costs differently if cost of each iteration is not roughly the same
- **Nested loops**
  - **Total cost is number of iterations or the outer loop times the cost of the inner loop**
  - **same caution as above**

Cost of operations: **Method Calls**

- **Cost for calling a function (method) is cost of...**

  cost of evaluating the arguments (constant or non-constant)

  + cost of actually calling the function (constant overhead)

  + cost of passing each parameter (normally constant time in Java for both numeric and reference values)

  + cost of executing the function body (constant or non-constant?)

```
System.out.print(lineNumber);
System.out.println("Answer is " + Math.sqrt(3.14159));
```

- **Note that "evaluating" and "passing" an argument are two different things**

Exercise

Analyze the running time of printMultTable

Pick the problem size

Count the number of steps

```
// print multiplication table with
// n rows and columns
void printMultTable(int n) {
    for (int k=0; k <=n; k++) {
        printRow(k, n);
    }
}
```

```
// print row r with length n of a
multiplication table
void printRow(int r, int n) {
    for (int k = 0; k <= r; k++) {
        System.out.print(r*k + " ");
    }
    System.out.println( );
}
```


Analysis

Comparing Algorithms

- **Suppose we analyze two algorithms and get these "times" (numbers of steps):**
  - **Algorithm 1: $37n + 2n^2 + 12$**
  - **Algorithm 2: $50n + 42$**

  How do we compare these? What really matters?
- **Answer: In the long run, the thing that is most interesting is the cost as the problem size n gets large**
  - **What are the costs for n=10, n=100; n=1,000; n=1,000,000?**
  - **Computers are so fast that how long it takes to solve small problems is rarely of interest**

Orders of Growth

- What happens as the problem size doubles?

N	$\log_2 N$	5N	N $\log_2 N$	N^2	2^N
8	3	40	24	64	256
16	4	80	64	256	65536
32	5	160	160	1024	~10^9
64	6	320	384	4096	~10^{19}
128	7	640	896	16384	~10^{38}
256	8	1280	2048	65536	~10^{76}
10000	13	50000	10^5	10^8	~10^{3010}

Sidebar: How Many Atoms In Earth?

As estimated by Tuquynh Nguyen (CSE143, Summer 2003):

"The obvious answer would be: More than any of us can count! But after doing a bit of researching myself, I found that you can only estimate the fractional amount of the most abundant types of atoms that the Earth consists of, and then with the molecular weight and Advocagro's number, we can determine the number of atoms. This seems to be the general breakdown:

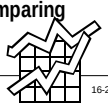
Iron: 1.26(10^51) atoms
Oxygen: 1.08(10^51) atoms
Silicon: 5.40(10^50) atoms
Magnesium: 4.68(10^50) atoms
Sulfur: 7.20(10^49) atoms
Calcium: 3.60(10^49) atoms
Aluminum: 3.60(10^49) atoms"

SUM: 3.49(10^51) TOTAL ATOMS IN THE WORLD

Compare that to the number 10^{3010} on the preceding chart. !!!

Asymptotic Complexity

- Asymptotic: Behavior of complexity function as problem size gets large
  - Only thing that really matters is higher-order term
  - Can drop low order terms and constants
- The asymptotic complexity gives us a (partial) way to answer "which algorithm is more efficient"
  - Algorithm 1: $37n + 2n^2 + 120$ is proportional to n^2
  - Algorithm 2: $50n + 42$ is proportional to n
- Graphs of functions are handy tool for comparing asymptotic behavior



Big-O Notation

- Definition: If f(n) and g(n) are two complexity functions, we say that

  $f(n) = O(g(n))$ (pronounced f(n) is O(g(n)) or is order g(n))

  if there is a constant c such that

  $f(n) \leq c \cdot g(n)$

  for all sufficiently large n

Exercises

- Prove that $5n+3$ is $O(n)$
- Prove that $5n^2 + 42n + 17$ is $O(n^2)$

Implications

- The notation f(n) = O(g(n)) is *not* an equality
- Think of it as shorthand for
  - "f(n) grows at most like g(n)" or
  - "f grows no faster than g" or
  - "f is bounded by g"
- O( ) notation is a *worst-case* analysis
  - Generally useful in practice
  - Sometimes want *average-case* or *expected-time* analysis if worst-case behavior is not typical (but these are often harder to analyze)

Complexity Classes

- Several common complexity classes (problem size n)
  - Constant time: O(k) or O(1)
  - Logarithmic time: O(log n) [Base doesn't matter. Why?]
  - Linear time: O(n)
  - "n log n" time: O(n log n)
  - Quadratic time: $O(n^2)$
  - Cubic time: $O(n^3)$
  - …
  - Exponential time: $O(k^n)$
- $O(n^k)$ is often called *polynomial time*

Rule of Thumb

- If the algorithm has polynomial time or better: practical
  - typical pattern: examining all data, a fixed number of times
- If the algorithm has exponential time: impractical
  - typical pattern: examine all combinations of data
- What to do if the algorithm is exponential?
  - Try to find a different algorithm
  - Some problems can be proved not to have a polynomial solution
  - Other problems don't have known polynomial solutions, despite years of study and effort
  - Sometimes you settle for an approximation

    The correct answer most of the time, or an almost-correct answer all of the time

Big-O Arithmetic

- For most common functions, comparison can be enormously simplified with a few simple rules of thumb
- Memorize complexity classes in order from smallest to largest: O(1), O(log n), O(n), O(n log n), $O(n^2)$, etc.
- Ignore constant factors

  $300n + 5n^4 + 6 + 2^n = O(n + n^4 + 2^n)$
- Ignore all but highest order term

  $O(n + n^4 + 2^n) = O(2^n)$

Analyzing List Operations (1)

- We can use O( ) notation to compare the costs of different list implementations
- Operation — Dynamic Array — Linked List
  - Construct empty list
  - Size of the list
  - isEmpty
  - clear

Analyzing List Operations (2)

- Operation — Dynamic Array — Linked List
  - Add item to end of list
  - Locate item (contains, indexOf)
  - Add or remove item once it has been located

Wait! Isn't this totally bogus??

- Write better code!!
  - More clever hacking in the inner loops

    (assembly language, special-purpose hardware in extreme cases)
- Moore's law: Speeds double every 18 months
  - Wait and buy a faster computer in a year or two!

- But …

How long is a Computer-Day?

- If a program needs f(n) microseconds to solve some problem, how big a problem can it solve in a day?
  - One day = 1,000,000*24*60*60 = $9*10^{10}$ (aprox)

f(n)	n such that f(n) = one day
n	$9 * 10^{10}$
5n	$2 * 10^{10}$
$n \log_2 n$	$3 * 10^{9}$
n^2	$3 * 10^{5}$
n^3	$4 * 10^{3}$
2^n	36

Speed Up The Computer by 1,000,000

- Suppose technology advances so that a future computer is 1,000,000 fast than today's

f(n)	original n	speedup on future machine
n	$9 * 10^{10}$	million times larger
5n	$2 * 10^{10}$	million times larger
$n \log_2 n$	$3 * 10^{9}$	60,000 times larger
n^2	$3 * 10^{5}$	1,000 times larger
n^3	$4 * 10^{3}$	100 times larger
2^n	36	+20

Practical Advice For Speed Lovers

- First pick the right algorithm and data structure
  - Implement it carefully, insuring correctness
- Then optimize for speed – but only where it matters
  - Constants do matter in the real world
  - Clever coding can speed things up, but the result is likely to be harder to read, modify
  - Use tools to find hotspots – concentrate on these

"Premature optimization is the root of all evil"
– Donald Knuth

More Advice...

"It is easier to make a correct program efficient than to make an efficient program correct"

-- Edsgar Dijkstra

Summary

- Analyze algorithm sufficiently to determine complexity
- Compare algorithms by comparing asymptotic complexity
- For large problems, an asymptotically faster algorithm will always trump clever coding tricks
- Optimize/tune only things that actually matter, once you've picked the best algorithm

Computer Science Note

- Algorithmic complexity theory is one of the key intellectual contributions of Computer Science
- Typical problems
  - What is the worst/average/best-case performance of an algorithm?
  - What is the best complexity bound for all algorithms that solve a particular problem?
- Interesting and (in many cases) complex, sophisticated math
  - Probabilistic and statistical as well as discrete
- Still some key open problems
  - Most notorious: P ?= NP