CSE 143

Highlights of Tables and Hashing

Tables: Ch. 11, pp.515-522

Hashing: Ch. 12 pp.598-604

Looking Up Data

- A common pattern in many programs is to look up data
  - Find student record, given ID#
  - Find phone #, given person's name
  - A CS example: a compiler's "symbol table"
    `look up identifier to find its type, location, etc.`
- Because it is so common, many data structures for it have been investigated
  - We could use arrays, linked lists, general trees, binary search trees, etc.
  - Could also step back and consider an abstract data type for looking things up: a Table

Table Terminology

- ***Key***: Portion of pair used to look up data, like an index (aka *domain* value)
- ***Value***: Portion of pair that contains data (aka *range* value)

aTable:

Key	Value
"4476542K"	23,440
"3828122E"	27,640
"24601JVJ"	15,203
"994802WE"	45,210
"8675309A"	28,776

Key (employee ID) **Value** (salary)

Table ADT

- Characteristics of *table* ADT type
  - Set of key/value pairs
  - No duplicate keys
- Operations on tables
  - *Retrieve* value given key
  - *Insert* value at key
  - *Delete* key and associated value from table
- Uses:
  - Phone book, class roster, book index, databases
- Sometimes also called a *dictionary* or *map*

Example of Operations

```
// phone book example: name is key, phone# is
 value

Table pb;

pb.insert("Sarah", 5552345);
pb.insert("Richard", 3450011);
pb.insert("Bart", 6661212);
int bartsNumber = pb.retrieve("Bart");
pb.remove("Richard");
```


Structures for Efficient Retrieval

- In many Table applications, *retrieve* is the most common operation
  - So retrieve needs to be efficient
- Many different data structures are possible
  - Array (sorted or unsorted?), List, Binary tree, Binary search tree. But not… stack, queue
  - Footnote: Some languages (Perl, Javascript) contain a built-in "associative array" construct
- With a sorted array and binary search, *retrieve* would be O(log N)
  - Same for binary search tree *find* (assuming reasonably balanced tree)

Can we do better than O(log N)?

- Answer: Yes … sort of, if we're lucky.
- General idea: *take the key of the data record you're inserting, and use that number directly as the item number in a list (array).*
- Example:
  - Assume you want quick access to a table of your friends. All of them have unique social security numbers (in the range 000-00-0000 to 999-99-9999).
  - If you had an array with 1,000,000,000 elements, each friend could be instantly located in the array by their social security number.
  - *What's wrong with the above scheme?*

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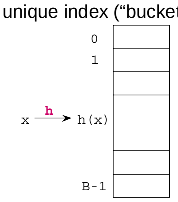
Hash Functions

- Basic idea:
  - *Don't* use the key value directly.
  - Given an array of size B, use a *hash function*, h(x), which maps the given record key x to some (hopefully) unique index ("bucket") in the array.



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Hashing and Tables

- Hashing gives us another implementation of Table ADT
- Find (retrieval) algorithm: Hash the key; this gives an index; use it to find the value stored in the table
- If this scheme worked, it would be O(1)
  - Great improvement over Log N.
- Main problems
  - Finding a good hash function
  - Collisions
  - Wasted space in the table

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An Apparent Sidetrack

- Problem to solve: *Given a list of n integers, determine if there is a pair of duplicate values*

37591	31576	64085	42782	25475	70900	79953	76186
67887	84848	81309	30822	77867	45852	65289	8322
79367	40520	58053	16030	34723	22116	41073	60522
34399	31616	85965	82102	73707	38316	153	11282
7623	61416	10741	46686	73123	69780	65105	21866
75567	5760	66525	80214	63835	48652	49593	42066
20055	16248	12213	35758	12147	13828	7729	2266
13263	57984	73181	34246	51755	58053	31817	52754
23863	9160	56677	62462	65715	68404	48097	66762
37519	52480	28045	68294	71131	6252	81689	51570
72119	71944	9797	77822	56563	67348	51553	86986
88303	10656	6925	89654	63099	25036	84393	47426

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Element Uniqueness: Two Solutions

1. Nested loop: for each element in the array, scan the rest of the array to see if there is a duplicate.
   - $O(n^2)$
2. Sort the data. Then scan array (once) for duplicates.
   - O(nlog n) time to sort, O(n) to scan.

- Anything simpler??
- Any solution that looks like hashing?

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Element Uniqueness (2)

- Step 1: Assign to buckets, based on value mod 100

37591	31576	64085	42782	25475	70900	79953	76186
67887	84848	81309	30822	77867	45852	65289	8322
79367	40520	58053	16030	34723	22116	41073	60522
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Element Uniqueness (3)

- Step 2: Look inside each bucket for duplicates

37591	31576	64085	42782	25475	70900	79953	76186
67887	84848	81309	30822	77867	45852	65289	8322
79367	40520	58053	16030	34723	22116	41073	60522
34399	31616	85965	82102	73707	38316	153	11282
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Hashing Functions

- The hash function we choose depends on the type of the key field (the key we use to do our lookup).
  - There are zillions of possible hash functions, but...
  - Finding a *good* hash function can be hard
- Example:
  - Student Ids (integers)

```
h(idNumber) = idNumber % B

eg. h(678921) = 678921 % 100 = 21
```

  - Names (char strings)

```
h(name) = (sum over the ascii values) % B

eg. h("Bill") = (66+105+108+108) % 100 = 87
```


Collisions

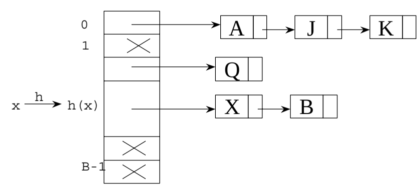
- Collisions occur when multiple items are mapped to same location

```
h(idNumber) = idNumber % B
h(678921) = 21
h(354521) = 21
```

- Issues
  - Relative size of table to number of data items
  - Choice of hash function
- With a bad choice of hash function we can have lots of collisions
- Even with a good choice of hash functions there may be some collisions

Collision Resolution

- One strategy: *Bucket hashing* (aka open hashing)
- Each cell (bucket) in the array is the head of a *linked list* of items:



Many other solutions have been studied

Analysis of hash table ops

- *Insert* is easy to analyze:
  - It is just the cost of calculating the hash value O(1), plus the cost of inserting into the front of a linked list O(1)
- *Retrieve* and *Delete* are harder. To do the analysis, we need to know:
  - The number of elements in the table (N)
  - The number of buckets (B)
  - The quality of the hash function

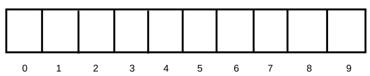
Hashing Analysis (2)

- We'll assume that our hash function distributes items evenly through the table, so each bucket contains N/B items (N/B is called the *load factor*)
- On average, doing a lookup or a deletion is O(N/B) (Which is O(1), if N/B is constant)
- Using a good hash function and keeping B large with respect to N, we can guarantee constant time insertion, deletion, and lookup
- *Note that this means growing (rehashing) the hash table as more items are inserted.*

Open vs. Closed Hashing

- Open hashing uses linked lists for the buckets
- What is closed hashing??
- All data is stored in the array
  - If there is a collision, the next available cell is used
  - Avoids overhead of linked lists and works well in practice
  - Example: hash the following into a table of size 10

```
17, 23, 47, 52, 71, 86, 63, 96
```



Dynamic Hashing

- Another implementation concept
- As number of stored records increases, dynamically increase the number of buckets
- Brute force: make a new (larger) array and copy (rehash) all the data to it
- More subtle implementations are also possible

Hashing and Files

- We've spoken of the hashed data as being stored in an array (in memory)
- Hashing is also very appropriate for disk files
- Efficient look-up techniques for disk data are essential
  - Disks are thousands of times slower than memory
  - Even a LogN look-up algorithm is too slow for a database application!
- **Many structures we have studied (linked lists, trees, etc.) do *not* scale well to large disk files**
  - **Hashing does scale well**

Four Drawbacks to Hashing

- Finding a good hash function
  - Risk of bad behavior
- Dealing with collisions
  - Simplest method: use linked list for buckets
- Wasted space in the array
  - Not a big deal if memory is cheap
- Doesn't support ordering queries (such as we would want for a real dictionary)

Summary

- Hash tables are specialized for Table (Dictionary) operations: *Insert, Delete, Lookup*
- Principle: **Turn the key field of the record into a number, which we use as an index for locating the item in an array.**
- O(1) in the ideal case; less in practice
- Problems: collisions, wasted space
- Implementations: open hashing, closed hashing, dynamic hashing
- Highly suitable for database files, too