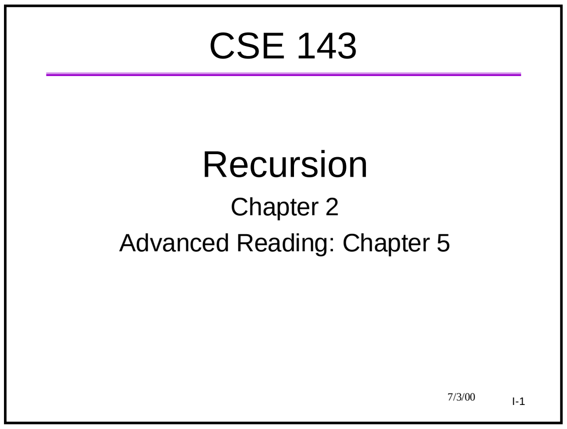

CSE 143

Recursion

Chapter 2

Advanced Reading: Chapter 5

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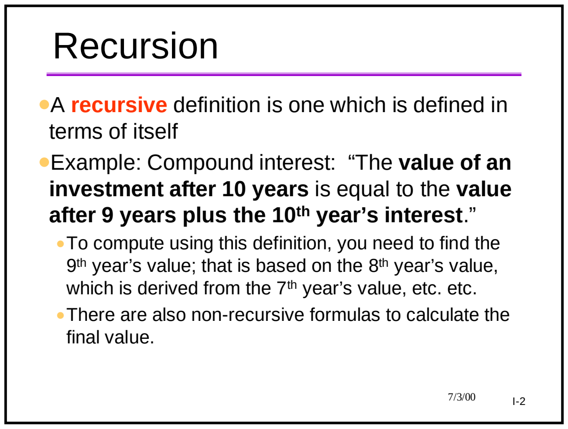

Recursion

- A **recursive** definition is one which is defined in terms of itself
- Example: Compound interest: "The **value of an investment after 10 years** is equal to the **value after 9 years plus the 10th year's interest**."
  - To compute using this definition, you need to find the 9th year's value; that is based on the 8th year's value, which is derived from the 7th year's value, etc. etc.
  - There are also non-recursive formulas to calculate the final value.

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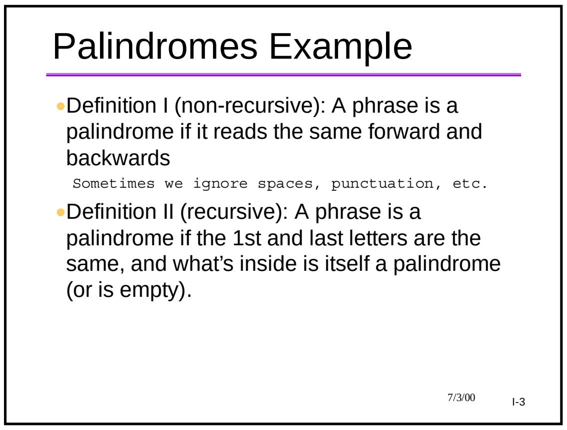

Palindromes Example

- Definition I (non-recursive): A phrase is a palindrome if it reads the same forward and backwards

  `Sometimes we ignore spaces, punctuation, etc.`
- Definition II (recursive): A phrase is a palindrome if the 1st and last letters are the same, and what's inside is itself a palindrome (or is empty).

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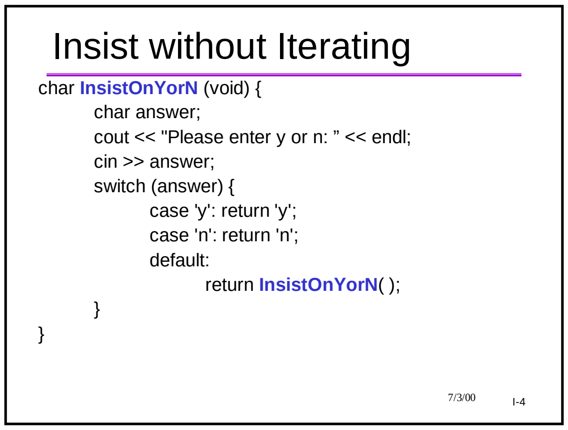

Insist without Iterating

```
char InsistOnYorN (void) {
        char answer;
        cout << "Please enter y or n: " << endl;
        cin >> answer;
        switch (answer) {
                case 'y': return 'y';
                case 'n': return 'n';
                default:
                        return InsistOnYorN( );
        }
}
```

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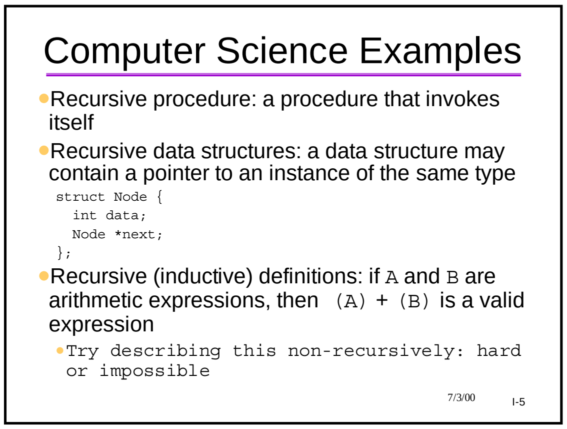

Computer Science Examples

- Recursive procedure: a procedure that invokes itself
- Recursive data structures: a data structure may contain a pointer to an instance of the same type

```
struct Node {
  int data;
  Node *next;
};
```

- Recursive (inductive) definitions: if `A` and `B` are arithmetic expressions, then `(A) + (B)` is a valid expression
  - `Try describing this non-recursively: hard or impossible`

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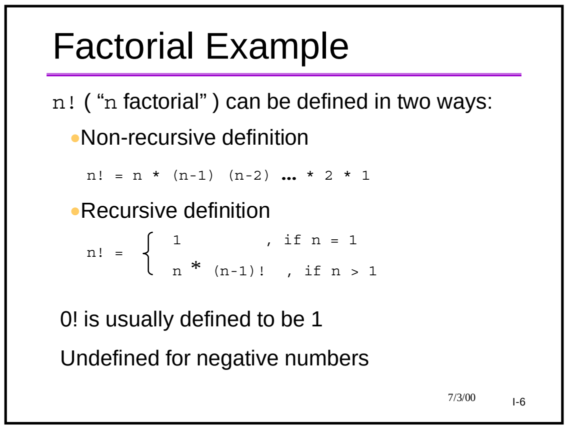

Factorial Example

`n!` ("`n` factorial") can be defined in two ways:

- Non-recursive definition

  $$n! = n * (n-1)\ (n-2) \dots * 2 * 1$$
- Recursive definition

  $$n! = \begin{cases} 1 & \text{, if } n = 1 \\ n * (n-1)! & \text{, if } n > 1 \end{cases}$$

0! is usually defined to be 1

Undefined for negative numbers

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Factorial (2)

- How do we write a function that reflects the recursive definition?

```
int factorial(int n) {
   assert(n >= 1);
   if ( n == 1 )
      return 1;
   else
      return n * factorial(n-1);
}
```

- The `factorial` function invokes itself.
- How can this work?

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How Recursion Works: Simplified Model

- ***Every time you call a function, you get a fresh copy of it.***
  - **If you call recursively, you end up with more than one copy of the function active**
- ***When you exit a function, only that copy of it goes away.***
- In reality...
  - there's only one copy of the code (instructions), but separate copies of the data (variables and parameters)

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How Recursion Works: Details

- Review: local variables and formal params are
  - allocated when { } block is entered,
  - deleted when block is exited.
- Here's how:
  - Whenever a function is called (or { } block is entered), a new "activation record" is created, containing:
    -- a separate copy of all local variables and parameters
    -- control info, such as where to return to
  - Activation record is alive until the function returns
    Then it is destroyed
  - This applies *whether or not* function is recursive!

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Tracing the Process

- To trace function calls
  - draw a box each time a function is called.
  - draw an arrow from caller to called function
  - label data (local vars, params) inside the box
  - indicate the returned value (if any)
  - cross out the box after return
    and don't reuse it!
- Question: how is this different from a "static call graph"?
- Note that *no* special handing is needed just because a function happens to be recursive!

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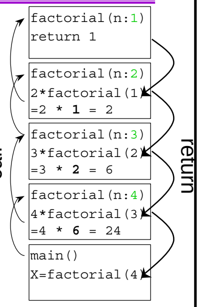
Trace Example

```
int factorial(int n) {
  if ( n == 1 )
    return 1;
  else
    return n * factorial(n-1);
}

...
int main (void) {
  int x = factorial(4);
  cout << "4! = " << x << endl;
...
```



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What is Recursion?

- A programming technique
  - a function calling itself
- An approach to problem-solving
  - Look for smaller problems similar to the larger problem
- A way of thinking about algorithms
  - Turns out to lead to good mathematical analyses
- The natural algorithmic technique when recursive data structures are involved

Recursion takes practice
  - Eventually it becomes a natural habit of thought

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What About Efficiency??

- Is recursion faster/slower/smarter/more powerful etc. than iteration? We'll talk about that, too -- later
- Learning *how* to drive a car, vs learning *when and where* to drive a car.
  - Different kinds of knowledge
  - The first especially requires focused practice

Infinite Recursion

- **Mathematically:**
  - **n! = n * (n-1)! = (n-1)! * n**
  - **Why not program it as (n-1)! * n in that order?**

```
int BadFactorial(n) {
  int x = BadFactorial(n-1);
  if ( n == 1 )
     return 1;
  else
     return n * x;
}
```

- What is the value of `BadFactorial(2)`?
- The rule: Must always have some way to make recursion stop, otherwise it runs forever:

Using Recursion Properly

- For correct recursion (recursion that does something useful and eventually stops), need two parts:
  1. One or more ***base cases*** that are not recursive
     `if ( n == 1 ) return 1; // no recursion in this case`
  2. One or more ***recursive cases*** that operate on *smaller* problems that get *closer* to a base case

     return n * **factorial(n-1)**;

     //factorial(n-1) is a smaller problem than factorial (n)
- The base case(s) should **always** be checked before the recursive calls

Searching an Array

- Problem statement: Given an array `A` of `N` `int`s, search for an element with value `x`
- First, an iterative solution: "Linear Search"

```
// Return index of x if found, or -1 if not

int Find (int A[], int N, int x)
{
  for ( int i = 0; i < N; i++ )
    if ( A[i] == x )
      return i;
  return -1;
}
```

- How efficient is Linear Search?
  - Might find x on first step, or you might have to check all N values
  - On average, it takes about N/2 times through the loop

Binary Search

- If array is *sorted*, we can search faster
  - Start search in middle of array
    `if x is right there in the middle, you're done`
  - If `x` is less than middle element, need to search only in lower half
  - If `x` is greater than middle element, need to search only in upper half
  - continue the seach within the half chosen
- Why is this faster than linear search?
  - At each step, linear search throws out *one* element
  - Binary search throws out *half* of remaining elements
- Why is recursion natural here?

Example

Find 26 in the following sorted array:

```
1 3 4 7 9 11 15 19 22 24 26 31 35 50 61
                 ↑
                    22 24 26 31 35 50 61
                              ↑
                    22 24 26
                        ↑
                          26
                          ↑
```

Binary Search (Recursive)

```
int BSearch(int A[], int size, int x) {
  return findInRange(A, x, 0, size-1);
}

int findInRange(int A[], int x, int lo, int hi) {
   if (lo > hi) return -1;
   int mid = (lo+hi) / 2;
   if (x == A[mid])
     return mid;
   else if (x < A[mid])
     return findInRange(A, x, lo, mid-1);
  else
     return findInRange(A, x, mid+1, hi);
}
```

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Linear vs Binary Search

- Linear search is simpler to program
- But much slower, except for very small N.
- How many array elements are examined?
  Ex.: N = 1,000,000
  - Linear search: 500,000 on average
  - Binary search: only about **20 (!)**
- Mathematically: how many times do you have to cut N in half before you get down to 1?
  - That is, **$2^? = N$**
  - Solution: **$\log_2 N$**.

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Kick-off and Helper Functions

- Previous example illustrates a common pattern:
  - Top-level "kick-off" function
    - Not itself recursive
    - Starts the recursion going
    - Returns the ultimate answer
  - Helper function
    - Contains the actual recursion
    - May require additional parameters to keep track of the recursion. (Actually solving a more general problem.)
- Client programs only need call the kick-off function

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Recursion with Array Params

```
double sum (double iArray [ ], int from, int to) {
  //find the sum of all elements in the array between index "from" and index "to"
        if (from > to)
                return 0.0;
        return iArray[from] + sum (iArray, from+1, to);
}
//Client code:
double CashValues[200];
...
double total = sum (CashValues, 0, 199);
```

- Implemented without kick-off/helper structure
  - but probably would benefit from having it

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What does this function do?

```
int mystery (int x) {
   assert (x > 0);
   if (x == 1)
       return 0;
   int temp = mystery (x / 2);
   return 1 + temp;
}
```

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Recursion vs. Iteration, I

- When to use recursion?
  - Processing recursive data structures
  - “Divide & Conquer” algorithms:
    1. Divide problem into subproblems
    2. Solve each subproblem recursively
    3. Combine subproblem solutions
- When to use iteration instead?
  - Nonrecursive data structures
  - Problems without obvious recursive structure
  - Problems with obvious iterative solution
  - Functions with a large "footprint"
    - especially when many iterations are needed

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Recursion vs. Iteration, II

- In theory, recursion and iteration are completely equivalent: any iteration can be rewritten using recursion, and vice-versa
- Recursion often gives simpler code
- Iteration is often more efficient
  - somewhat faster, less memory (but more code)
- A compromise:
  - If the problem is naturally recursive, design the algorithm recursively first
  - Later convert to iteration if needed for efficiency
  - General principle: "Make it right, then make it efficient"

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Dueling Factoids

- ***Factoid 1***: Some programming languages provide *no* iteration control statements!
  - loops must be implemented through recursion
  - rely on the compiler to make it efficient
  - Prolog, pure LISP
- ***Factoid 2***: Not all programming languages support recursion!
  - COBOL, FORTRAN (at least early versions)
  - Many highly paid programmers *never* use recursion

    `So... why do we make` *`you`* `do it??`

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Summary

- Recursion is something defined in terms of itself
- Activation records make it work
- Elements of recursive functions
  - Base case(s)
  - Recursive case(s)

    `Base case always checked first`
- When to use/when to avoid

As the course unfolds, we'll see more and more cases where recursion is natural to use

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