
CSE 142

Introduction to Recursion

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Outline for Today

- Review
 - Method calls and scope
 - Static methods
- Today
 - Recursion – methods that call themselves
 - Recursive and base cases
 - Implementation in Java

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Method Calls and Static Methods (Review)

- Recall that a static method is one that is associated with a class, not a particular instance of a class
- Often used for computations that are not naturally associated with some object

```
public class Math {  
    /** return the square root of x */  
    public static double sqrt(double x) { ... }  
    /** return the trigonometric sin of theta */  
    public static double sin(double theta) { ... }  
}
```

- Use

```
double sqrt2 = Math.sqrt(2.0);
```

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Recursive Definitions

- **Classic example: factorial**
- **Mathematical definition**

$$n! = \begin{cases} 1 & n \leq 1 \\ n * (n-1)! & \textit{otherwise} \end{cases}$$

- **Example**

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Factorial in Java

- Could write a loop to multiply $1 * 2 * 3 * \dots * n$
- Can also use the recursive definition directly!

```
/* return n! = 1 * 2 * 3 * ... * n */  
public static int factorial(int n) {  
  
  
  
  
  
  
}
```

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Trace

- **Evaluate**
factorial(4)

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How Can This Possibly Work?

- This is an example of a *recursive* method call – a method that calls itself as part of its implementation
- There is nothing really new here. A method call works as it always does:
 - First, allocate a new scope for the method's parameters and local variables
 - Second, initialize parameters with method call arguments
 - Third, begin execution of the method body
- Recursive methods work exactly the same
- Also works fine for non-static methods

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Method Call Trace

- Evaluate factorial(4)

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Recursive and Base Cases

- A recursive definition always has two parts
 - One or more *recursive* cases where the method calls itself
 - One or more *base* cases that return a result without an additional recursive call
- Rules
 - There *must* be at least one base case
 - Each recursive case *must* make progress towards reaching a base case
- Forgetting either one of these rules is a common source of errors in recursive methods
 - In particular, "infinite" recursion – never reaching a base case; each call generates yet another recursive call

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Towers of Hanoi

- Classic problem
- Setup
 - Three pegs
 - Set of disks of different diameters, initially on one peg with disks stacked in order – largest on bottom, smallest on top
- Problem: move all of the disks from the initial peg to one of the other two, without ever placing a larger disk on top of a smaller one
- Can you think of an algorithm to do this?
 - Hint: recursion *is* your friend

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Algorithm for Towers of Hanoi

- Your algorithm here

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Demonstration

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Iteration vs Recursion

- Turns out that any iterative algorithm can be reworked as a recursive algorithm, and vice versa
 - Use recursive calls wherever “looping” is needed
- Sometimes this is straightforward – e.g., factorial
- Sometimes less obvious – how would you implement towers of Hanoi iteratively?
 - A non-recursive solution to a naturally recursive problem often requires extra bookkeeping to keep track of what’s been done already and what needs to be done

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When to Use Recursion

- Recursion is a natural fit for problems that...
 - ... have one or more simple cases with a straightforward non-recursive solution (base cases)
 - ... have other cases that can be redefined as simpler versions of the original problem, and repeating these redefinitions gets closer to one of the simple cases (recursive cases)
- Take advantage of recursion when the problem matches
 - (Usually – there are occasions where a naturally recursive implementation is too slow or has too much overhead)

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Example: QuickSort

- Supposed we are asked to sort a list
- QuickSort is an very fast algorithm that makes use of recursion

```
/* Sort the items in list[from] to list[to] */
public void QuickSort(int from, int to) {
    if (to - from <= 1) {
        return; // base case - at most one item; nothing to do
    } else {
        pick some element x from list[from] to list[to];
        rearrange the list so that x is in position mid, and all items in list[from] to
        list[mid-1] are <= x and all items from list[mid+1] to list[to] are > x;
        QuickSort(from, mid-1);
        QuickSort(mid+1, to);
    }
}
```

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QuickSort Example

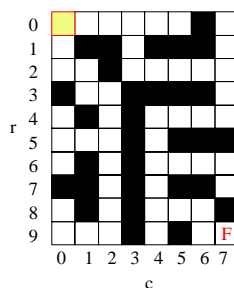
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Another Problem – Path Planning

- Idea: want to discover if there is a path from square at 0, 0 to square labeled F (which could be anywhere)
- Black squares represent obstacles
- Unless a path is blocked, can move up, down, left, or right
- Can you design an algorithm for this?
 - Hint: can you use recursion to help?
- Answer next time



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