This exam is closed book, closed notes. You have 50 minutes to complete it. It contains 8 questions and 7 pages (including this one), totaling 100 points. Before you start, please check your copy to make sure it is complete. Turn in all pages, together, when you are finished. Write your initials on the top of ALL pages (in case a page gets separated during test-taking or grading). Please write neatly; we cannot give credit for what we cannot read.

Good luck!
1 Short answer

1. (6 points) State the steps you should follow when writing a function. Write just one phrase for each step. (These steps come after you have defined the problem and decomposed it into parts, and before you run tests.) Depending on how you break up the tasks, you might write 3, 4, or 5 steps; you can get full credit for any of these answers as long as your steps include all the necessary parts and no extraneous parts.

   (a) ______________________
   (b) ______________________
   (c) ______________________
   (d) ______________________
   (e) ______________________

2. (6 points) Name the two types of documentation, and state for whom each one is intended.

   (a) ______________________
   Intended for: _______________________________________________________

   (b) ______________________
   Intended for: _______________________________________________________

3. (8 points) State three advantages of splitting up your code into functions. Give reasons that are as different as possible. Write no more than one sentence each (a phrase each is enough).

   (a) ______________________
   __________________________

   (b) ______________________
   __________________________

   (c) ______________________
   __________________________
2 Environment diagram

4. (12 points) Consider the following code:
```python
def exp(base, exponent):
    """Return base to the exponent power.
    Exponent is a non-negative integer.""
    if exponent == 0:
        return 1
    if exponent % 2 == 0:
        return exp(base*base, exponent/2)
    return base * exp(base, exponent - 1)
```

Here are the powers of 5, for your information:

- $5^0 = 1$
- $5^1 = 5$
- $5^2 = 25$
- $5^3 = 125$
- $5^4 = 625$
- $5^5 = 3125$
- $5^6 = 15625$

Given the expression $\text{exp}(5, 6)$, show the stack frames when the multiplication on the last line of code (the multiplication with the box around it) is about to be executed for the first time.

What are the two arguments to the multiplication referenced in the last question?

________________________  __________________________
3 Testing

5. (24 points) Consider the following function specification:

```python
def indices_of(lst, elt):
    """Return a sorted list of the indices at which the element appears in lst."""
```

Here is an example test:

```python
assert indices_of([3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3], 9) == [5, 12, 14]
```

A good test suite contains tests that are different from one another.
Apply the “0–1–many” principle to the `indices_of` function. Write three different tests, and state what is varying between them.

What is varying: 

Test 1: 

Test 2: 

Test 3: 

Now, use the “0–1–many” principle again, but apply it to something different. Again, write three different tests, and state what is varying between them.

What is varying: 

Test 1: 

Test 2: 

Test 3: 

Write an implementation of `indices_of`, in a single line of code.
Hint: use a list comprehension.
4 Write code

6. (12 points) Write the body of the following function.

```python
def word_frequency(list_of_tuples):
    """Return a dictionary mapping words to numbers.
    The input is a list of (word, number) tuples.
    In the output dictionary, each word maps to the sum of all numbers it
    co-occurs with in the input list.""
```

Here is a test case, to help you understand the specification:

```python
assert word_frequency([("to", 2), ("be", 2), ("or", 2), ("not", 3), ("to", 2),
                       ("be", 2)]) == {'be': 4, 'not': 3, 'or': 2, 'to': 4}
```
7. (16 points) Write a class `RatNum` that represents rational numbers. A rational number is the ratio of two integers; examples are $\frac{1}{2}$ and $\frac{22}{7}$. You do not have to write any documentation strings.

Once your class is defined, then running the following code:

```python
one_half = RatNum(1,2)
one_third = RatNum(1,3)
five_sixths = one_half.add(one_third)
print(one_half.to_float())
print(one_third.to_float())
print(five_sixths.to_float())
```

should print

```
0.5
0.333333333333
0.833333333333
```

Hint: Do not define any more methods than necessary.

Hint: Recall from middle school that $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.

Hint: You do not have to deal with client errors, such as using zero as the denominator.

Hint: Do not think about reducing fractions to lowest terms. That is not necessary for this problem. (If you don’t understand this hint, then don’t worry about it.)
8. (16 points) Write the body of the following function. Write the function twice: iteratively (using a loop) and recursively (using a recursive call). You may solve the two parts of this problem in either order. You may use the function \texttt{max}(x, y) that takes two numbers and returns the larger one.

## Iterative version
```python
def list_max(numbers):
    """Returns the largest value that appears in the argument list.
    The argument is a non-empty list of numbers.""
```

## Recursive version
```python
def list_max(numbers):
    """Returns the largest value that appears in the argument list.
    The argument is a non-empty list of numbers.""
```